On-the-job Training and On-the-job Search: Wage-Training Contracts in a Frictional Labor market

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INTRODUCTION

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Motivation

- Human Capital Accumulation (post schooling)
  - the major contributor individual wage growth and economic growth
  - the first best outcome: Workers should pay for the cost of general training.
  - In reality,
    - only firms can provide general training in many cases and
    - workers cannot commit to staying with the training firms

- This paper studies the coexistence of On-the-job Training and Search
  - Do productive firms provide more training?
  - Do firms provide the efficient level of training?
  - Do firms provide more training, as search friction is mitigated?
ILLUSTRATIVE EXAMPLES

- Becker (1964): Perfect Competition

\[ w_2 = y + f(x^B) \]
\[ \pi_2 = 0 \]
\[ w_1 = y - x^B \]
\[ \pi_1 = 0 \]

- Under perfect competition, the firm provides the efficient level of training, and the worker pays the training cost through lower wage during training.
Illustrative Examples

worker:

firm:

period 1

worker: $w_1 = y - x - w_1$

firm: $y - x - w_1$

period 2

worker: $w_2$

firm: $y + f(x) - w_2$

- Acemoglu (1997): exogenous job-turnover shock

\[ w_2 = \phi(y + f(x^A)) \]

\[ \pi_2 = (1 - \alpha)(1 - \phi)(y + f(x^A)) \]

\[ \pi^p = \alpha(1 - \phi)(y + f(x^A)) \]

\[ x^A \in \text{arg max} -x + (1 - \alpha)f(x) \]

\[ w_1 = \phi(y - x^A) \]

\[ \pi_1 = (1 - \phi)(y - x^A) \]

- positive externality for subsequent poaching firms (free rider problem)
Previous Literature

  - no on-the-job search by unskilled workers
  - no skilled unemployed workers
  - no productivity differential

- Fu (2011)
  - incorporates the piece rate sharing rule into Burdett and Mortensen (1998)
  - ends up with inefficient level of training

- Sanders and Taber (2012)
  - over-investment on job specific human capital
  - under-investment on general human capital
On-the-job Training and On-the-job Search

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The Basic Model

- Burdett and Mortensen (1998) with productivity differentials
- a unit measure of risk neutral (lifetime income maximizing) workers
  - A newly born worker enters the labor market as unskilled and unemployed.
  - The unemployed worker gets employed at rate $\lambda^0$.
  - The employed worker finds another job at rate $\lambda^1$ and gets laid off at rate $\delta$.
  - The employed worker acquires (general) skills at rate $\mu x$ through training.
  - All workers retire at rate $\rho$ and they are replaced with newly born workers.
- a unit measure of heterogenous firms ($p \sim H(p)$)
  - Each firm maintains one vacancy at every instant.
  - The recruiting firm with $p$ posts $(E_u(p), E_s(p)) = ((w_u(p), x(p), E_s^t(p)), (w_s(p)))$.
  - It meets an employed searcher at rate $\lambda^1$ and unemployed searcher at rate $\lambda^0$.
- $\varepsilon$-measure of noise firms
  - They offer only skilled wages from $\hat{F}_n : [\underline{p} + s, \bar{p} + s] \rightarrow [0, 1]$. 

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**The Basic Model**

- Unemployed Workers
  - retire at rate $\rho$, and get employed at rate $\lambda^0$.

  $$rU_i = b - \rho U_i + \lambda^0 \int \max\{z - U_i, 0\} dF_i(z), \text{ for each } i \in \{u, s\}$$

- Skilled Employed Workers
  - retire at rate $\rho$, get laid off at rate $\delta$, and find offers at rate $\lambda^1$.

  $$rE_s(p) = w_s - \rho E_s(p) + \delta(U_s - E_s(p)) + \lambda^1 \int \max\{z - E_s(p), 0\} dF_s(z)$$

- Unskilled Employed Workers
  - retire at rate $\rho$, get laid off at rate $\delta$, find offers at rate $\lambda^1$, and
  - acquire (general) skills at rate $\mu x$.

  $$rE_u(p) = w_u - \rho E_u(p) + \delta(U_u - E_u(p)) + \lambda^1 \int \max\{z - E_u(p), 0\} dF_u(z) + \mu x(E_s - E_u(p))$$
THE BASIC MODEL

- Operating Firms with Skilled Matches
  - deliver the committed values through ...

\[ rJ_s(p) = p + s - w_s(p) - [\rho + \delta + \lambda^1(1 - F_s(E_s(p)))]J_s(p) \]

- Operating Firms with Unskilled Matches
  - deliver the committed values through ...

\[ rJ_u(p) = \max_{w_u, x, E_s^t} p - w_u - c(x) - [\rho + \delta + \lambda^1(1 - F_u(E_u(p)))]J_u(p) + \mu x (J_s(p) - J_u(p)) \]

subject to the promise-keeping constraint on \( E_u(p) \).

- F.O.C.
  - \( w_s^t(p) = p + s \)
  - \( c'(x) = \mu (E_s^t(p) - E_u(p) - J_u(p)) \)
  - The promise keeping constraint determines unskilled wages.
THE BASIC MODEL

- Recruiting Firms
  - post \((E_u(p), E_s(p))\) to maximize

\[
[\lambda^0 u_s + \lambda^1 G_s(E_s)]J_s(E_s, p) + [\lambda^0 u_u + \lambda^1 G_u(E_u)]J_u(E_u, p)
\]
The Basic Model

Given firms’ productivity distribution $H(p)$, a steady state equilibrium with on-the-job training and on-the-job search consists of value equations $\{U_i, E_i, J_i\}$ compensation packages $\{(w_u(p), x(p), E_s^t(p)), (w_s(p))\}$ and steady state measures $\{F_i, G_i, u_i\}$ that jointly satisfy the following conditions.

(i) Given $F_i$, workers make optimal job turnover decision.

(ii) Given $\{F_i, E_i\}$, operating firms optimally deliver the committed values.

(iii) Given $\{G_i, u_i\}$, recruiting firms post their contract to maximize their profit.

(iv) $\{F_i, G_i, u_i\}$ are stationary and consistent with the behavior of each agent.
Steady State Equilibrium

Figure 1: Equilibrium Support of Wages
Steady State Equilibrium

- Proposition 1 The optimal training intensity is characterized by

\[
\frac{c'(x(p))(r + \rho + \delta)}{\mu} + x(p)c'(x(p)) - c(x(p)) = s + \delta(U_s - U_u) + \lambda^1 \int_{E_t^s(p)} [z - E_s^t(p)]dF_s(z) - \lambda^1 \int_{E_t^u(p)} [z - E_u(p) - J_u(p)]dF_u(z)
\]

- In particular, \(x(\bar{p}) < x(p)\) for any \(p \in [\underline{p}, \bar{p}]\) if and only if

\[
\int_{E_t^s(p)} [z - E_s^t(p)]dF_s(z) > \int_{E_t^u(p)} [z - E_u(p) - J_u(p)]dF_u(z)
\]
Efficiency Benchmarks

- Constrained Social Planner
  - maximizes the present value of the expected output flow throughout the life of a newly born worker in the steady state equilibrium.

\[
(r + \rho) S_s^*(p) = p + s + \delta(U_s^* - S_s^*) + \lambda^1 \int_{\bar{p}} [S_s^*(p') - S_s^*(p)] dH(p')
\]

\[
(r + \rho) S_u^*(p) = p - c(x^*(p)) + \delta(U_u^* - S_u^*) + \mu x^*(p)(S_s^*(p) - S_u^*(p))
\] + \lambda^1 \int_{\bar{p}} [S_u^*(p') - S_u^*(p)] dH(p')

\[
(r + \rho) U_i^*(p) = b + \lambda^0 \int_{\bar{p}} [S_i^*(p') - U_i^*(p)] dH(p')
\]

- chooses the training intensity such that

\[
c'(x^*(p)) = \mu(S_s^*(p) - S_u^*(p))
\]

\[
(\text{vs} \quad c'(x(p)) = \mu(E_s^t(p) - E_u(p) - J_u(p)))
\]
Efficiency Benchmarks

- Proposition 2 The training intensity in the social planner’s problem is characterized by

\[ c'(x^*(p))(r + \rho + \delta)/\mu + x^*(p)c'(x^*(p)) - c(x^*(p)) = s + \delta(U_s^* - U_u^*) \]

- In particular, \( dx^*/dp = 0, dx^*/d\lambda^1 = 0 \), and \( dx^*/d\lambda^0 > 0 \)
Efficiency Benchmarks

- In the market equilibrium,

\[ c'(x(p))(r + \rho + \delta)/\mu + x(p)c'(x(p)) - c(x(p)) = s + \delta(U_s - U_u) \]

\[ + \lambda^1 \int_{E_s^t(p)} [z - E_s^t(p)]dF_s(z) - \lambda^1 \int_{E_u(p)} [z - E_u(p) - J_u(p)]dF_u(z) \]

- In the social planner’s problem,

\[ c'(x^*(p))(r + \rho + \delta)/\mu + x^*(p)c'(x^*(p)) - c(x^*(p)) = s + \delta(U_s^* - U_u^*) \]
Efficiency Benchmarks

Figure 2: Training Intensity
On-the-job Training and On-the-job Search

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**Baseline Simulation**

Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\bar{p}, \overline{\bar{p}}]$</td>
<td>$[0.75, 1.75]$</td>
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<tr>
<td>$\eta$</td>
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<tr>
<td>$s$</td>
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<tr>
<td>$\gamma$</td>
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<tr>
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<tr>
<td>$\delta$</td>
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</tr>
<tr>
<td>$\lambda^0$</td>
<td>1.35</td>
</tr>
<tr>
<td>$\lambda^1$</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Productivity Distribution: $H(p) = \frac{1 - (p/\bar{p})^\eta}{1 - (\bar{p}/\overline{\bar{p}})^\eta}$

Cost function: $c(x) = x^\gamma$
Baseline Simulation

Figure 3: The Baseline Simulation Result I
Baseline Simulation

(a) In the Alternative Benchmark Setting

(b) In the Baseline Simulation

Figure 4: Training Intensity

<table>
<thead>
<tr>
<th></th>
<th>unskilled workers</th>
<th>skilled workers</th>
<th>training cost</th>
<th>total output</th>
<th>Net output</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>0.6001</td>
<td>0.3492</td>
<td>0.0159</td>
<td>1.3859</td>
<td>1.3700</td>
</tr>
<tr>
<td>PP</td>
<td>0.6027</td>
<td>0.3467</td>
<td>0.0156</td>
<td>1.3862</td>
<td>1.3707</td>
</tr>
<tr>
<td>ME/PP</td>
<td>0.9958</td>
<td>1.0073</td>
<td>1.0191</td>
<td>0.9997</td>
<td>0.9995</td>
</tr>
</tbody>
</table>

ME: the market equilibrium outcome  PP: the planner’s solution  ME/PP: the ratio of ME to PP

Table 2: The Outcome of the Baseline Simulation
COMPARATIVE STATICS

Figure 5: Training Intensity
COMPARATIVE STATICS

**Figure 6: Training Intensity**
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  - the first best outcome: Workers should pay for the cost of general training.
  - In reality,
    - only firms can provide general training in many cases and
    - workers cannot commit to staying with the training firms

- This paper studies the coexistence of On-the-job Training and Search
  - Hump-shaped training intensity
  - over-intensified general training
  - Mitigating search friction intensifies training but improves net output.
Thanks for listening!

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