Quantized volatility model for transaction data
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Price Movements

Random walk hypothesis

\[ P_t = P_{t-1} + w_t = P_0 + \sum_{j=1}^{t} w_j, \]

where \( w_j \) is a random variable,
\( E(w_j) = 0, \)
\( E(w_j w_k) = 0 \ (j \neq k), \) and
\( E(w_j w_j) = \text{constant} \)

Fama, E (1965) “Random walks in stock market prices”
Samuelson, P (1965) “Proof that properly anticipated prices fluctuate randomly”
Price Movements

A random walk hypothesis

\[ \log(P_t) = \log(P_0) + \sigma B_t + \left( \mu + \frac{\sigma^2}{2} \right) t \]

where \( B_t \) is a wiener process, \( \mu_t \) is a drift rate, and \( \sigma_t \) is a volatility.


A non-random walk hypothesis

\[ P_t = \mu_t dt + P_{t-1} + \sigma_t dB_t \]

Price Movements

Short-term seasonality vs long-term stable volatility

Random walk hypothesis

Vs

Non-random walk hypothesis
Price Movements

Price Movements with ticks

\[ P_t = P_{t-1} + \varepsilon_t \]

where

\[ \varepsilon_i = \pm \varepsilon_0 \times i, \]

\( \varepsilon_0 \) is the minimum size of price increment specified by the stock exchange, \( i \) is an integer.
Price Movements

Price Movements with ticks

\[ P_t = P_{t-1} + \varepsilon_t \]

<table>
<thead>
<tr>
<th>売気配</th>
<th>買気配</th>
</tr>
</thead>
<tbody>
<tr>
<td>成行</td>
<td></td>
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<td>331</td>
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<td>17130</td>
</tr>
<tr>
<td>即時約定指値注文</td>
<td>17125</td>
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</tbody>
</table>

Price increments

Uniform Distribution
Price Movements

Price Movements with ticks

\[ P_t = P_{t-1} + \varepsilon_t \]

<p>| | | |</p>
<table>
<thead>
<tr>
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</tr>
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<tbody>
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<td>買気配</td>
</tr>
<tr>
<td>成行</td>
<td>361</td>
<td>16555</td>
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<tr>
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<td>296</td>
<td></td>
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<td>16540</td>
<td>369</td>
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</table>
| 時刻 | 現在値 | 前回比 | 出来数
| 17:55 | 16550 | 0 | 4 |
| 17:55 | 16550 | 0 | 8 |
| 17:54 | 16550 | 0 | 1 |
| 17:54 | 16550 | 0 | 5 |
| 17:54 | 16550 | 0 | 6 |
| 17:54 | 16550 | 0 | 2 |
| 17:54 | 16550 | 0 | 80 |
Price Movements

Price Movements with ticks

\[ P_t = P_{t-1} + \varepsilon_t \]
The sum of squared price increments

Squared price increments

\[ e_i = (\pm \varepsilon_i)^2 \]

where \(\varepsilon_i = \pm \varepsilon_0 x_i\)

Sum of squared price increments

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

Sum of squared price increments

$$E = \sum_{i=1}^{I} N_i e_i$$

must be stable for a long-term.
The sum of squared price increments

Sum of squared price increments must be stable for a long-term.

But why?

Markets balance the interests between

Investors and market makers.
The sum of squared price increments

Sum of squared price increments must be stable for a long-term, but why?

Investors want to minimize the bid-ask spread and have homogeneous transaction price.

Market makers cover their losses from adverse selections.
The sum of squared price increments

Darkice, Iceberg algorithms and stealth trading strategies are implemented to reduce market impacts.

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

How to obtain the sum of squared price increments?

Remove all transactions without price movements and bid-ask bounce effects.

It is called an **immediacy trades**.

\[ E = \sum_{i=1}^{I} N_i e_i \]

In Nikkei 225 mini, 98% of transactions are not immediacy trades.
The sum of squared price increments

How does a price of immediacy trade move?

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

How does a price of immediacy trade move?

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

If prices of immediacy trade follow a random walk,
MMs and investors prefer stable markets.

Use the runs test and the Durbin-Watson test.

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

The runs test p-value

The probability of p-value > 0.1 is 0.59.

From 2016.01 to 2017.04 (hourly analysis)

$$E = \sum_{i=1}^{I} N_i e_i$$
The sum of squared price increments

The Durbin-Watson test

Average $dw=2.3$

From 2016.01 to 2017.04 (hourly analysis)

$$E = \sum_{i=1}^{l} N_i e_i$$
The sum of squared price increments

Immediacy trades may follow a random walk process.

$E = \sum_{i=1}^{I} N_i e_i$
The sum of squared price increments

How the sum of squared price increments moves?

The average sspi=27.01

from 2015.08 to 2015.09

\[ E = \sum_{i=1}^{l} N_i e_i \]
The sum of squared price increments

How the sum of squared price increments moves?

The average sspi = 26.1  from 2016.01 to 2017.04

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

How the sum of squared price increments moves?

From 2015.08 to 2015.09
from 2016.01 to 2017.04

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments might be stable for a long-term, but have a seasonality in a short-term.

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

Vs

The realized volatility (1 hour interval)

from 2016.01 to 2017.04

Corr = 0.62

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

The number of immediacy trades

Vs

The realized volatility (1 hour interval)

rom 2016.01 to 2017.04

Corr=0.84

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

The number of immediacy trades

Vs

The sspi (1-hour interval)

from 2016.01 to 2017.04

Corr = 0.87

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

Close price – Open price in session

Vs

The difference between no. up ticks and no. down ticks

from 2016.01 to 2017.04

Corr=0.98

\[ E = \sum_{i=1}^{I} N_i e_i \]
The sum of squared price increments

Conclusions

1. Transaction prices may follow a random walk process.
   1. Market makers prefer the markets that the price movements are stable over time. Thus it is easy for them to cover the losses from adverse selections.
   2. Investors prefer the trades that minimize the market impacts.
2. The sum of squared price increments is fixed where the market makers and investors interest could be balanced.
3. Is it reasonable to analyze risky asset markets based on a financial return?

\[ E = \sum_{i=1}^{I} N_i e_i \]