Forecasting with DSGE Models: Theory and Practice

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Introduction

- Estimated DSGE models are now widely used for
  - empirical research in macroeconomics;
  - quantitative policy analysis and prediction at central banks.
- The use of DSGE models at central banks has triggered a strong interest in their forecast performance.
- Talk is based on a chapter for *Handbook of Economic Forecasting*
- Goals:
  - Review recent advances in forecasting with DSGE models.
  - Present some innovations in regard to the incorporation of external information and the use of DSGE-VARs

Bird's Eye View of the Literature

- Comparison of DSGE forecasts (RMSE, $\ln \det$ of forecast error covariance, log score) with other forecasts
  - AR, VAR, BVAR, DFM, etc.
  - professional forecasts: SPF, Bluechip, Green(Teal)book
- Incorporating external information into the DSGE model forecasts
- Projections conditional on alternative instrument-rate paths
- Forecasting with hybrid models
- Combination of forecasts from different DSGE models as well as other models

Outline of Presentation

- DSGE model used throughout this talk
- Data for forecast evaluation
- Benchmark Forecasts
- Using External Information
- Conditioning on alternative instrument-rate paths
- Forecasting with DSGE-VARs

DSGE Model

- Small-scale model New Keynesian DSGE model (e.g., Woodford, 2003)
  - Euler equation, NK Phillips curve, monetary policy rule
  - 3 exogenous shocks: technology, government spending, monetary policy

- Keep in mind monetary policy rule:

$$R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp[\sigma_R \epsilon_{R,t}], \quad R_{*,t} = (r_* \pi_*) \left(\frac{\pi_t}{\pi_*} \right)^{\psi_1} \left(\frac{Y_t}{\gamma_* Y_{t-1}} \right)^{\psi_2}.$$

- Measurement equations:

$$\begin{aligned} GDP(GR)_t &= 100 \ln \gamma_* + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \\ INFLATION_t &= 100 \ln \pi_* + \hat{\pi}_t \\ FEDFUNDS_t &= 100(\ln r_* + \ln \pi_*) + \hat{R}_t. \end{aligned}$$

- Note: Handbook chapter will also contain results for large-scale model

Generating Forecasts with a DSGE Model

- DSGE Model = State Space Model
  - Measurement Eq:

$$y_t = \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_2(\theta)s_t$$

  - State Transition Eq:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t$$

- Posterior distribution of DSGE model parameters:

$$p(\theta|Y_{1:T}) = \frac{p(Y_{1:T}|\theta)p(\theta)}{p(Y_{1:T})}, \quad p(Y_{1:T}) = \int p(Y_{1:T}|\theta)p(\theta)d\theta.$$

- Objective of interest is predictive distribution:

$$p(Y_{T+1:T+H}|Y_{1:T}) = \int p(Y_{T+1:T+H}|\theta, Y_{1:T})p(\theta|Y_{1:T})d\theta.$$

- Use numerical methods to generate draws from predictive distribution.

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Data Sets

- Forecast evaluations are pseudo-out-of-sample.
- In the past, forecast evaluations were based on the latest available data vintage at the time the study was conducted.
- More recently: use of real time data sets. Particularly important if DSGE model forecasts are compared to professional forecasts.

Reference: Croushore and Stark (2001)

Real Timing

Quarter	Greenbook Date	End of Estimation Sample T	Initial Forecast Period $T+1$
Q1	Jan 21	2003:Q3 (F)	2003:Q4
	Mar 10	2003:Q4 (P)	2004:Q1
Q2	Apr 28	2003:Q4 (F)	2004:Q1
	June 23	2004:Q1 (P)	2004:Q2
Q3	Aug 4	2004:Q2 (A)	2004:Q3
	Sep 15	2004:Q2 (P)	2004:Q3
Q4	Nov 3	2004:Q3 (A)	2004:Q4
	Dec 8	2004:Q3 (P)	2004:Q4

Notes: (A) denotes *advance* NIPA estimates, (P) refers to *preliminary*, and (F) to *final* NIPA estimates.

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Evaluation

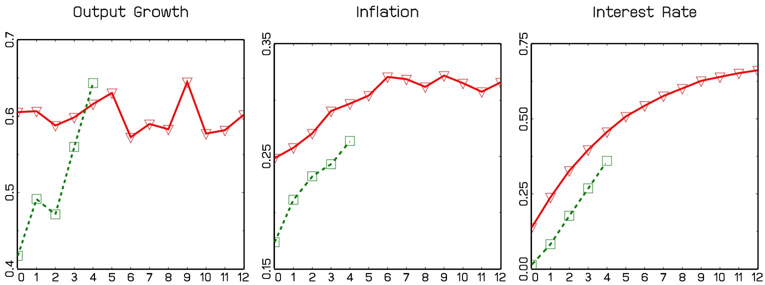
- Most common approach: evaluation of univariate point forecasts under a quadratic loss function
- Root-Mean-Squared Forecast Errors (RSMEs):

$$RMSE(h) = \sqrt{\frac{1}{T_{max} - T_{min}} \sum_{\tau=T_{min}}^{T_{max}} \left(y_{\tau+h} - \hat{y}_{\tau+h|\tau}\right)^2}.$$

- Begin of estimation sample: 1983:Q1
- We use Greenbook dates:
  - T_{min}: January 1992
  - T_{max}: December 2009

References: Edge and Gürkaynak (2010)

RMSEs of Benchmark Forecasts



- RMSEs for QoQ rates in percentages
- Red, Solid = DSGE Model
- Green, Dotted = Blue Chip

RMSEs in the Literature

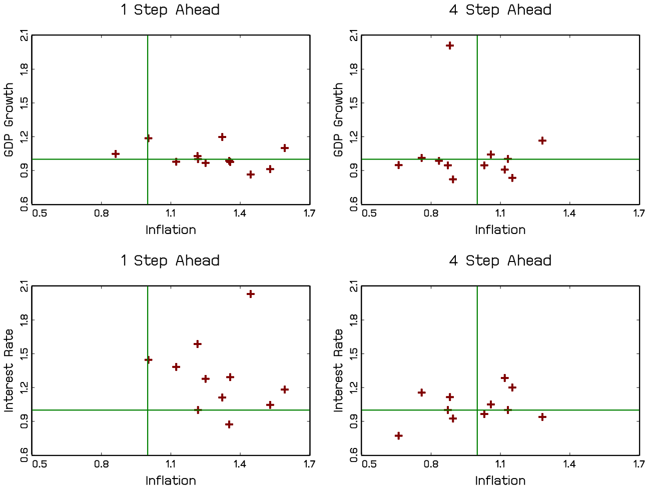


Figure depicts RMSE ratios: DSGE (reported in various papers) / AR(2) (authors calculation).

Evaluation

- Even though DSGE forecasts are potentially RMSE dominated by other forecasts one can ask: are predictive densities are well calibrated?

- Roughly: in a sequential forecasting setting events that are predicted to have 20% probability, should roughly occur 20% of the time.

- Probability Integral Transforms:
  - If Y is cdf $F(y)$, then

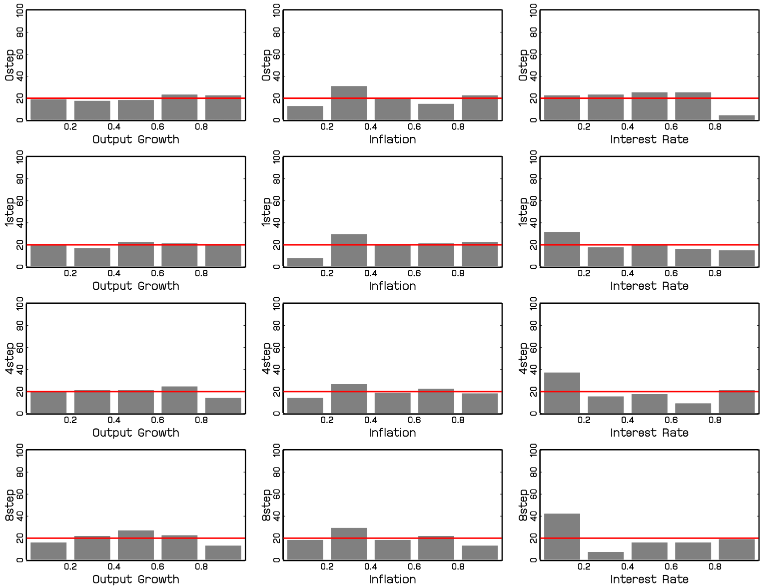
$$\mathbb{P}\{F(Y) \leq z\} = \mathbb{P}\{Y \leq F^{-1}(z)\} = F\big(F^{-1}(z)\big) = z$$

  - PITs

$$z_{i,t,h} = \int_{-\infty}^{y_{i,t+h}} p(\tilde{y}_{i,t+h} | Y_{1:T}) d\tilde{y}_{i,t+h}.$$

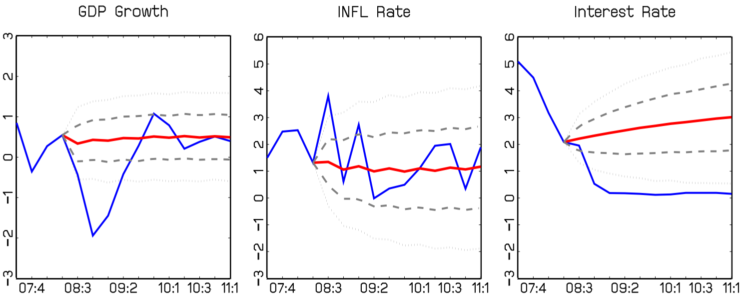
References for PITs: Rosenblatt (1952), Dawid (1984), Kling and Bessler (1989), Diebold, Gunther, and Tay (1998), Diebold, Hahn, and Tay (1999), ..., Geweke and Amisano (2010), Herbst and Schorfheide (2011).

Benchmark Forecasts: Histograms for PITs

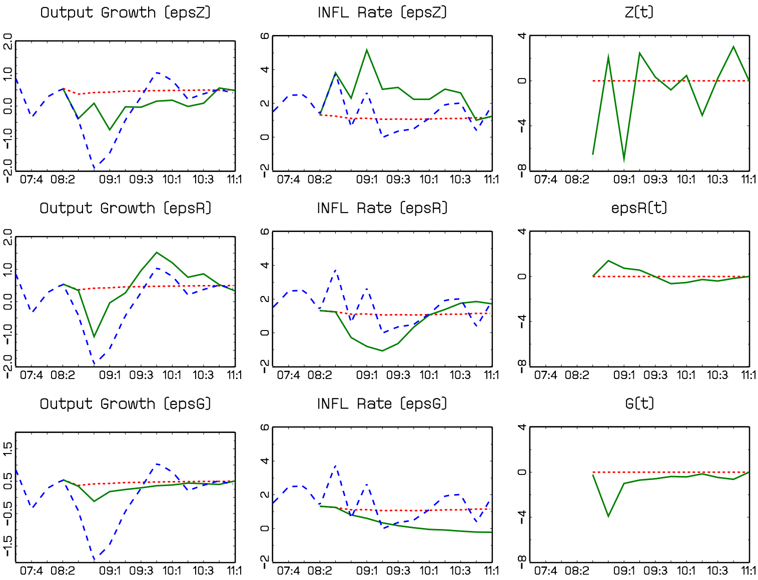


Predicting the Crisis: Interval Forecasts and Actuals

September 2008



Contribution of Realized Shocks to Forecast Errors



Benchmark Forecasts – Summary

- DSGE model RMSEs are decent but not stellar. Clearly dominated over short horizons.
- Probability densities seem to be in line with empirical frequencies in our small model.
- Our DSGE model missed the big drop in output and interest rates in 2008.

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Utilizing External Information

- Professional forecasts beat DSGE forecasts in the short-run with respect to variables for which real time information is available.
- How can this information be incorporated?

1. Without modifying the DSGE model:

   replace $p(Y_{T+1:T+H}|\theta, Y_{1:T})$ by $p(Y_{T+1:T+H}|\theta, Y_{1:T}, Z_T)$

   Interpretation: forecaster (but not the agents!) obtains information about future realization of shocks as well as time T state of economy.

2. By introducing additional shocks – and possibly informational frictions – into the DSGE model.

Utilizing External Information without Model Modification

- *Approach 1:* true Y_{T+1} = external info Z_T + *noise*
  Roughly speaking, factorize
  $$p(Y_{T+1}, Y_{T+2:T+H}|\theta, Y_{1:T}) = \underbrace{p(Y_{T+1}|\theta, Y_{1:T})}_{\text{replace by } p(Y_{T+1}|\theta, Y_{1:T}, Z_T)} \times p(Y_{T+2:T+H}|\theta, Y_{1:T}, Y_{T+1})$$
- *Approach 2:* external info Z_T = true Y_{T+1} + *noise*
  For example, let $p(Z_T|Y_{T+1}, Y_{1:T}, Y_{T+2,T+H}) = p(Z_T|Y_{T+1})$ and use Bayes Theorem to determine $p(Y_{T+1:T+H}|\theta, Y_{1:T}, Z_T)$.
- Approaches are the same for hard conditioning: $noise = 0$.
- Under both approaches the forecaster essentially obtains information about the shocks ϵ_{T+1} as well as the initial state s_T.
- Treatment of parameters $p(\theta|Y_{1:T})$ versus $p(\theta|Y_{1:T}, Z_T)$.

References: Waggoner and Zha (1999), Benes, Binning, and Lees (2008), Andersson, Palmqvist, and Waggoner (2010), Maih (2010)

Illustration 1

- Hard conditioning on Bluechip nowcasts.
- Use Kalman-filter/smoother to extract information about future shocks provided by external information;
- Generate draws from predictive distribution conditional on the extracted information about future shocks

Illustration 1 – Greenbook Forecast Dates, Estimation Samples, and External Nowcasts in 2004

Greenbook Date	End of Estimation Sample T	External Nowcast		Initial Forecast
		BlueChip Date	Period $T+1$	Period $T+2$
Jan 21	2003:Q3	Jan 10	2003:Q4	2004:Q1
Mar 10	2003:Q4	Mar 10	2004:Q1	2004:Q2
Apr 28	2003:Q4	Apr 10	2004:Q1	2004:Q2
June 23	2004:Q1	June 10	2004:Q2	2004:Q3
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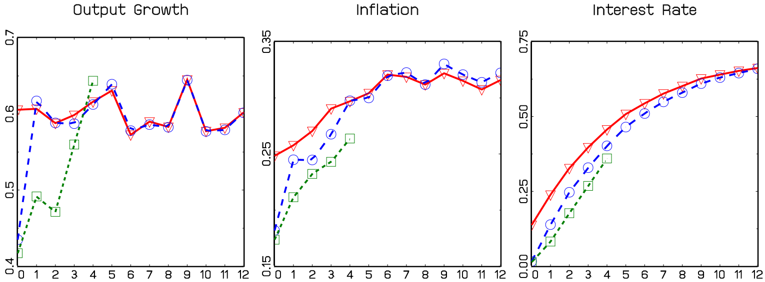
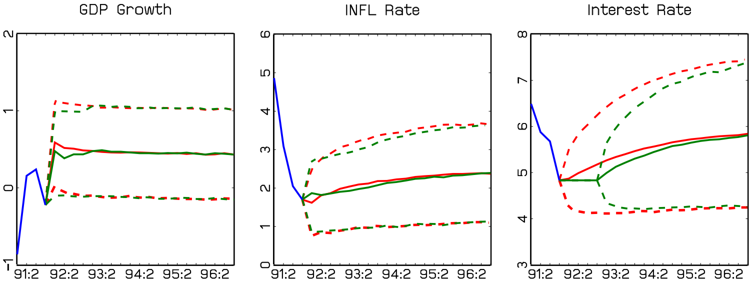
Nov 3	2004:Q3	Nov 10	2004:Q4	2005:Q1
Dec 8	2004:Q3	Dec 10	2004:Q4	2005:Q1

Illustration 1 – RMSE Comparison: Benchmark versus Model with Ext. Nowcasts



- Red, Solid = DSGE Model
- Blue, Dashed = DSGE Model + Blue Chip Nowcasts
- Green, Dotted = Blue Chip

Illustration 2 – Suppose in January 1992 We Hard-Condition on the Belief that Interest Rates Stay Constant for 1 Year



Period	ϵ^g	ϵ^R	ϵ^z
$t = T + 1$	-0.10	-0.10	-0.07
$t = T + 2$	-0.24	-0.04	-0.02
$t = T + 3$	-0.19	-0.04	-0.01
$t = T + 4$	-0.17	-0.02	-0.01

Remarks

- The likelihood of the structural shocks that are needed to attain the path of observables implied by the external information provides a measure of how plausible this external information is in view of the model. References: Benes, Binning, and Lees (2008)
- Rather than using the Kalman filter, one could select a subset of shocks that guarantee that pre-specified future values of observables are attained, e.g., use MP shocks to implement an interest rate path. More on this later... References: Smets and Wouters (2004), Christoffel, Coenen, and Warne (2007)
- Nonparametric approaches for soft and hard conditioning. References: Robertson, Tallman, Whiteman (2005), Herbst and Schorfheide (2010)
- Incorporating monthly information into the estimation of DSGE models. Giannone, Monti, and Reichlin (2009)

External Information Is Observation of Agents' Information in Model

- Three approaches:
  1. Add measurement equation:

     $$\text{external info} = \text{agents' expectations } + \textit{noise}$$

  2. Introduce additional structural shocks such that agents' expectations and external information, e.g. survey forecasts, can be equated.
  3. External information is interpreted as a signal that agents in the model receive and can use to form expectations.
- We shall focus on the *second* approach:
  1. Example 1: incorporate long-run inflation expectations
  2. Example 2: incorporate interest-rate expectations

Related literatures: anticipated shocks, DSGE models with informational frictions

Using Long-Run Inflation Expectations

- Many countries have experienced a decline in the inflation rate
- Inflation forecasts are important for central banks
- In a constant π_* model the long-run forecast of inflation is essentially the sample average, which often implies an implausible reversion to a relatively high inflation rate.
- Possible solution: incorporate long-run inflation expectations.

References: Aruoba and Schorfheide (2010), Wright (2011)

Example 1: Long-Run Inflation Expectations

- Modify interest-rate feedback rule as follows:

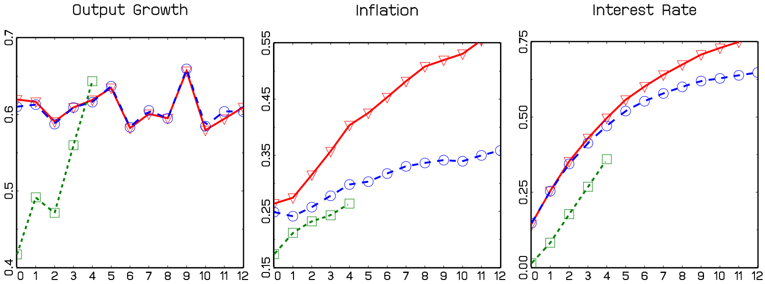
$$R_t = R_{*,t}^{1-\rho_R} R_{t-1}^{\rho_R} \exp[\sigma_R \epsilon_{R,t}], \quad R_{*,t} = (r_* \pi_{*,t}) \left(\frac{\pi_t}{\pi_{*,t}} \right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2}.$$

- $\pi_{*,t}$ is the time-varying target inflation rate.
- Assume that agents forecast the target according to

$$\pi_{*,t} = \pi_{*,t-1} + \sigma_\pi \epsilon_t^\pi.$$

- Long-run inflation expectations are interpreted as observations on $\pi_{*,t}$.
- To amplify the effect, we change the beginning of estimation sample from 1983:Q1 to 1965:Q1.

Example 1: RMSE Comparison: Benchmark versus Model with $\pi_{*,t}$



- Red, Solid = Benchmark DSGE Model
- Blue, Dashed = $\pi_{*,t}$ DSGE Model
- Green, Dotted = Blue Chip

Note: gain is a lot smaller if estimation sample starts in 1983:Q1.

Example 2: Using Interest Rate Expectations

- We've seen that the interest rate forecasts of the DSGE model are relatively poor.
- Not appealing from a policy maker's perspective.
- We'll use interest rate expectations $\mathbb{E}_T[R_{T+1}]$, $\mathbb{E}_T[R_{T+2}]$, $\ldots$ when generating forecasts.

Example 2: Using Interest Rate Expectations

- To match external interest rate forecast and model-implied expectations, add anticipated monetary policy shocks to the model.

- Simple version of the new policy rule can be expressed as:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\psi_\pi \hat{\pi}_t + \sigma_R \epsilon_t^R + \sum_{k=1}^{K} \sigma_{R,k} \epsilon_{k,t-k}^R,$$

  where ϵ_t^R is the usual unanticipated policy shock, and $\epsilon_{k,t-k}^R$, $k = 1, \ldots, K$ is an anticipated policy shock that affects the policy rule k periods later.

- So far, anticipated shocks are only used during forecasting step. We distributed the estimated variance of the MP shock across the unanticipated and two anticipated MP shocks.

Example 2: How Do Anticipated Policy Shocks Work?

- Simple Model:

$$
\begin{aligned}
y_t &= \mathbb{E}[y_{t+1}] - (R_t - \mathbb{E}[\pi_{t+1}]) \\
\pi_t &= \beta \mathbb{E}[\pi_{t+1}] + \kappa y_t \\
R_t &= \frac{1}{\beta}\pi_t + \epsilon^R_t + \epsilon^R_{1,t-1}.
\end{aligned}
$$

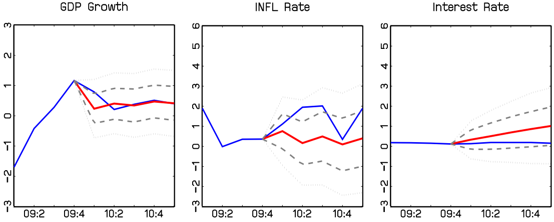
- Solution:

$$
\begin{aligned}
y_t &= -\psi\big(\epsilon^R_t + \epsilon^R_{1,t-1} + \psi\epsilon^R_{1,t}\big) \\
\pi_t &= -\kappa\psi\big(\epsilon^R_t + \epsilon^R_{1,t-1} + (\psi+\beta)\epsilon^R_{1,t}\big) \\
R_t &= \psi\big(\epsilon^R_t + \epsilon^R_{1,t-1} - \frac{1}{\beta}\kappa(\psi+\beta)\epsilon^R_{1,t}\big)
\end{aligned}
$$

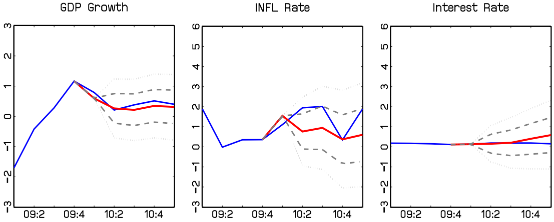
where $\psi = (1+\kappa/\beta)^{-1}$.

Density Forecasts with and without Interest Rate Expectations – March 2010

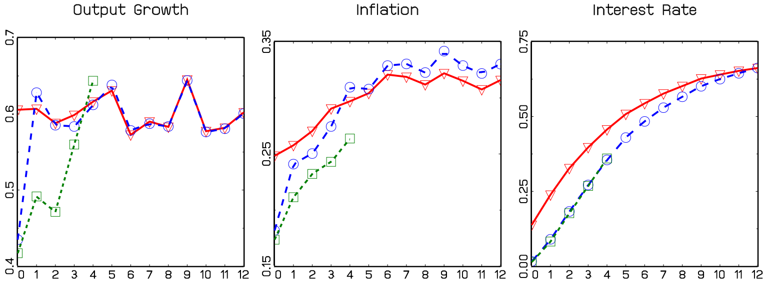
Benchmark DSGE Model



Using Nowcasts and Interest Rate Expectations

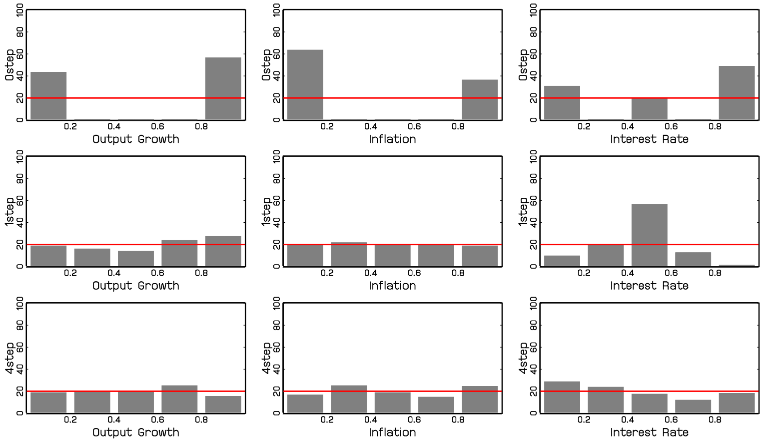


RMSE Comparison: Benchmark versus Model with Ext. Nowcasts and Interest Rate Expectations



- Red, Solid = Benchmark DSGE Model
- Blue, Dashed = DSGE Model with Interest Rate Expectations and Blue Chip nowcasts.
- Green, Dotted = Blue Chip

Histograms of PITs: Model with Ext. Nowcasts and Interest Rate Expectations



- Hard conditioning on nowcasts understates uncertainty in the current quarter.
- One-year ahead forecasts appear well calibrated

Using External Information – Summary

- Hard-conditioning on external nowcasts, which improved short-run forecasting performance of DSGE model, but understates uncertainty in predictive densities.
- We showed how time-varying inflation targets and anticipated monetary policy shocks can be used to incorporate inflation and interest rate expectations to improve forecasts.

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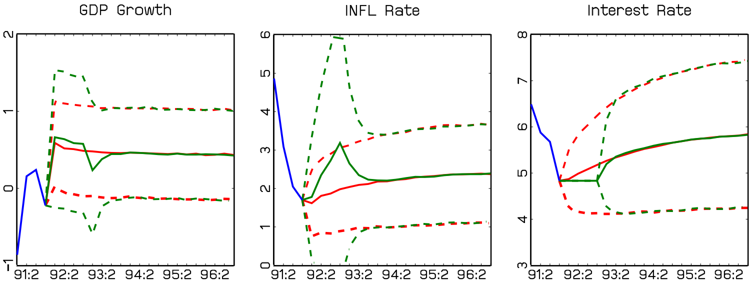
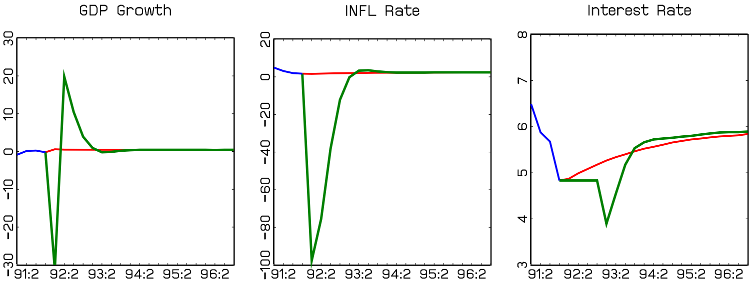
Projections Conditional on Alternative Instrument-Rate Paths

- Policy analysis is one of the potential strength of the DSGE model forecasting approach.
- Two approaches:
  1. Choose sequence of unanticipated policy shocks to attain the desired instrument-rate path. References: Leeper and Zha (2003), Smets and Wouters (2004).
  2. Choose sequence of anticipated policy shocks to attain the desired instrument-rate path. References: Laseen and Svenson (2009).
- Only the second approach is consistent with the notion that the central bank credibly announces an interest rate path.

Conditional Projections Using Unanticipated Shocks – Constant Interest Rate for 4 Periods



Conditional Projections Using Anticipated Shocks – Constant Interest Rate for 4 Periods



Conditioning on Alternative Instrument Paths – Summary

- The use of unanticipated MP shocks is not consistent with the notion of a policy that tries to announce interest rate paths.

- Anticipated MP shocks appear conceptually attractive but – depending on the properties of the estimated model – might generate fairly implausible predictions for non-policy variables.

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Hybrid Models

- Goal: relax some of the DSGE model restrictions to improve forecasting performance
- Hierarchical Hybrid Models, e.g. DSGE-VAR. References: Ingram and Whiteman (1994), Del Negro and Schorfheide (2004), Del Negro and Schorfheide (2009)
- Additive Hybrid Models, e.g. DSGE + Measurement Errors, DSGE + Generalized Trends, DSGE + DFM. References: Boivin and Giannoni (2006), Kryshko (2010), Schorfheide, Sill, and Kryshko (2010), Consolo, Favero, and Paccagnini (2009), Canova (2009)..
- Mixtures of Models. References: Wolters (2010), Waggoner and Zha (2011)

Hierarchical Hybrid Models: DSGE-VAR

- VAR

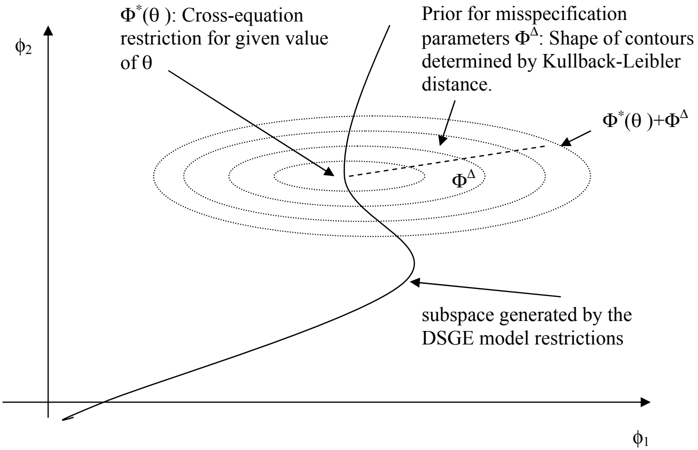
$$y_t = \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p} + \Phi_c + \Sigma_{tr}\Omega\epsilon_t,$$

with binding functions:

$$\Phi^*(\theta), \quad \Sigma^*(\theta), \quad \Omega^*(\theta).$$

- Allow for deviations from restriction functions...

Hierarchical Hybrid Models



DSGE-VARs

- Overall, the setup leads to

$$p_\lambda(Y, \Phi, \Sigma, \theta) = p(Y|\Phi, \Sigma) p_\lambda(\Phi, \Sigma, \Omega|\theta) p(\theta). \quad (1)$$

- In previous applications $\Omega|\theta$ is a pointmass centered at $\Omega^*(\theta)$
- Posterior for λ:

$$p(\lambda|Y) \propto (Y|\lambda) = \int p_\lambda(Y|\theta) p(\theta) d\theta. \quad (2)$$

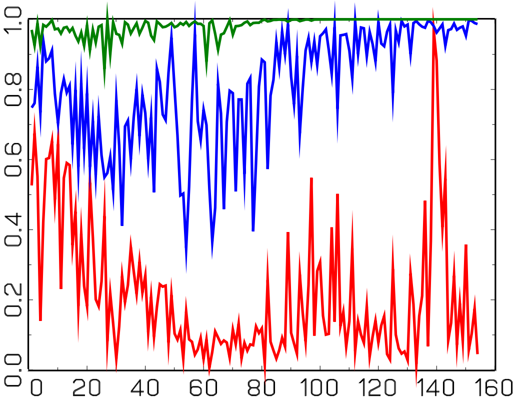
- Applications: model evaluation, forecasting, policy analysis.

References: Ingram and Whiteman (1994), Del Negro and Schorfheide (2004), Del Negro and Schorfheide (2009)

Application

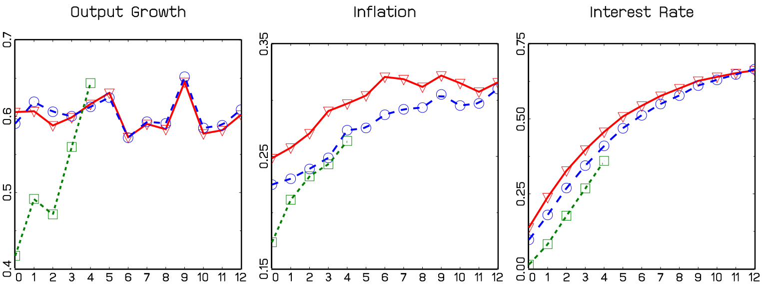
- *Prior* can be interpreted as *posterior* calculated from observations generated from DSGE model.
- We start out with a Minnesota prior and then "add" artificial observations from the DSGE model.
- $\lambda = 0$ means no DSGE model observations to construct the prior, $\lambda = \infty$ means that we impose DSGE restrictions $\Phi^*(\theta)$, $\Sigma^*(\theta)$, and $\Omega^*(\theta)$.
- Innovation here: (i) we mix Minnesota and DSGE model prior; (ii) average over hyperparameter λ.

Posterior Weights for λ



- Grid for λ: 0.1, 0.5, 1, 2, ∞
- Red $= \mathbb{P}_T\{\lambda \leq 0.1\}$
- Blue $= \mathbb{P}_T\{\lambda \leq 0.5\}$
- Green $= \mathbb{P}_T\{\lambda \leq 1\}$

RMSE Comparison: Benchmark versus DSGE-VAR



- Red, Solid = Benchmark DSGE Model
- Blue, Dashed = DSGE-VAR
- Green, Dotted = Blue Chip

Forecasting with DSGE-VARs – Summary

- DSGE model restrictions can be relaxed in many different ways, which opens the door for forecast improvements.
- DSGE-VAR approach creates a hybrid model that in many dimensions mimics the underlying DSGE model.
- In our illustration there are gains in forecast performance, but they are smaller than those attained by incorporating real time information.

Conclusion

- There exists a large literature on forecasting with DSGE models
- Most of the forecast evaluation focuses on RMSEs
- Some recent work on evaluating density forecasts and predictions comovements, e.g. Herbst and Schorfheide (2010).
- Lots to be gained from incorporating real time information. Models with anticipated shocks or informational frictions open interesting avenues to incorporate expectation data into forecasts.
- Projections conditional on interest rate paths still problematic.