The Canon Institute for Global Studies

CIGS Working Paper Series No. 14-003(E)

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*First version: May 2012. We would like to thank Noriyuki Yanagawa for valuable discussion. We gratefully acknowledge financial support from the Japan Society for the Promotion of Science (Kakenhi).

General Incorporated Foundation
The Canon Institute for Global Studies
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Phone: +81-3-6213-0550　　http://www.canon-igs.org

A macroeconomic model of liquidity crises*

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Abstract

We develop a simple macroeconomic model that captures key features of a liquidity crisis. During a crisis, the supply of short-term loans vanishes, the interest rate rises sharply, and the level of economic activity declines. A crisis may be caused either by self-fulfilling beliefs or by fundamental shocks. It occurs as a result of market failure due to debt overhang in short-term loans. The government's commitment to deposit guarantee reduces the likelihood of self-fulfilling crisis but increases that of fundamental crisis.

Keywords: Debt overhang, liquidity, systemic crisis.
JEL Classification numbers: E30, G01, G21.

*First version: May 2012. We would like to thank Noriyuki Yanagawa for valuable discussion. We gratefully acknowledge financial support from the Japan Society for the Promotion of Science (Kakenhi).

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1 Introduction

The global recession in the late 2000s has been the deepest economic downturn since the 1930s. As argued, for instance, by Lucas and Stokey (2011), it was a liquidity crisis that made the recession so severe, just as in the Great Depression.[1] A liquidity crisis is characterized as a sudden evaporation of the supply of liquidity, which leads to a large drop in production and employment.[2] Our objective in this paper is to provide a simple macroeconomic framework for understanding the mechanism behind such crises.

For this, we embed banks in an otherwise standard real business cycle framework in a way similar to, for instance, Gertler and Karadi (2011).[3] Banks collect deposits from households, and provide loans to firms. In terms of loans and deposits, the following three assumptions are crucial for our theory of liquidity crises. First, loans to firms and bank deposits are in the form of risky debt. Second, there are loans and deposits in different maturity, "short-term" and "long-term." Third, all loans have the same seniority regardless of their maturity. Thus the recovery rate for the short-term and long-term loans become identical. This also applies to deposits.

A liquidity crisis can be caused either by self-fulfilling beliefs or by shocks to fundamentals. The mechanism is very simple. Let us begin with a self-fulfilling crisis. Suppose that for some reason banks believe that firms are unable to repay their loans and go bankrupt. Given such beliefs, the short-term interest rate for loans goes up. If its increase is large enough, firms indeed go bankrupt justifying the beliefs of banks. Furthermore, massive defaults of firms may induce depositors to believe that their banks also go bankrupt. Then the interest rate on bank deposits also rises, which would cause banks to default as well. A fundamental crisis occurs when a bad enough productivity shock makes firms insolvent. Again, bankruptcy of firms leads to that of banks, and a liquidity crisis. Both types of the crisis result in an evaporation of the supply of short term loans, a rise in the short-term rates, and a decline in production and employment.

It is worth noting the importance of the three assumptions on loans and deposits discussed above. In each period, firms and banks are indebted with long-term debt, and thus, depending on the current revenue, they may go default. If their bankruptcies are expected, they would find it difficult to obtain short-term funds because in the case of default short-term creditors would be treated equally with long-term creditors. Thus, the mechanism underlying liquidity crises in our model is debt overhang in the sense that firms and banks are indebted too much to obtain new loans and deposits.[4] In particular, in our

[1] Overviews of the crisis are given by Adrian and Shin (2010), Brunnermeier (2009), and Gorton (2010), among many others.

[2] See, for instance, Borio (2009).

[3] See also Gertler and Kiyotaki (2010), Gertler, Kiyotaki, and Queralto (2012).

[4] Renegotiation between the lender and the borrower does not necessarily resolve the inefficiency of debt

model, both firms and banks face the debt-overhang problem, which reinforce each other.

In terms of government interventions, we consider the effects of a policy that guarantees bank deposits. We find that it has the following type of trade-offs. On the one hand, if the government commits to guarantee bank deposits, the possibility of self-fulfilling crises is reduced. On the other hand, however, it raises the probability of fundamental crises. The overall welfare effect of the bailout policy would therefore depend on the probabilities of self-fulfilling and fundamental crises.

In terms of the related literature, our model is closely related to the bank-run models such as Bryant (1980), Diamond and Dybvig (1983), Allen and Gale (1998), Uhlig (2010), Ennis and Keister (2009), Keister (2012), and Gertler and Kiyotaki (2012), among many others. In these models, a crisis occurs when there is a run on existing deposits. In our model, it occurs when there is an evaporation of short-term loans. Arguably, both aspects are present in actual liquidity crises. In this sense, we view the two approaches complementary.

Our model is also related to the literature on debt overhang, as discussed above. In particular, Philippon (2009, 2010) considers a two-period model in which banks and households suffer from debt overhang. Note, however, that unlike most models of debt overhang, our model is of infinite horizon, and hence, the degree of debt overhang is endogenously determined.[5]

The rest of the paper is organized as follows. In the next section we describe a simple example. The basic structure of the model economy is described in section 3. Then liquidity crises caused by self-fulfilling beliefs are considered in section 4; those due to fundamental shocks are in section 5; Some policy implications are discussed in section 6. Concluding remarks are given in section 7.

2 An example

A simple numerical example would also help to understand the key mechanism of our model.

2.1 Self-fulfilling crisis

Let us start with a self-fulfilling crisis. Consider a bank, a firm, and a depositor. Initially, the firm owes 50 to the bank and the bank owes 50 to the depositor. They are long-term debt of the firm and the bank. If the firm borrows 10 additionally as short-term loans,

overhang. For this, see, for instance, Albuquerque and Hopenhayn (2004).

[5] Occhino and Pescatori (2010) also analyze the debt-overhang distortion in an infinite horizon model, but in a different context. Their focus is on how it amplifies business cycles, while ours is on how it results in a systemic financial crisis.

then it can produce 70. If it fails, it only produces 30. To make the short-term loan, the bank needs to borrow 10 from the depositor. Both the depositor and the bank provide the short-term funds if the gross rate of return is unity. Then there exist two equilibria: "good" and "bad." The short-term loan is made in the good equilibrium, but not in the bad one. In the good equilibrium, all three agents expect that the gross market rate is one. The bank borrows 10 from the depositor and lends it to the firm. The firm produces 70 and pays (50+ 10 =) 60 to the bank. Then the bank pays back 60 to the depositor. In this case, the firm obtains 10 as a profit and the bank and the depositor are break-even.

In the bad equilibrium, on the other hand, the depositor believes that the bank will default on her debt and repay only $\frac{3}{5}$ of the total liability, and the bank believes that the firm will default on its debt and repay only $\frac{3}{5}$ of its total liability. Because the depositor believes that only $\frac{3}{5}$ of the claim to the bank asset is recoverable, he lends 10 to the bank only if the gross rate of interest is no less than $\frac{5}{3}$. Similarly, since the bank has to repay $10 \times \frac{5}{3}$ to the depositor if she borrows 10 and she also believes that only $\frac{3}{5}$ of the claim to the firm's asset is recoverable, she lends 10 to the firm only if the gross rate of interest is no less than $(\frac{5}{3} \times \frac{5}{3} =)\ \frac{25}{9}$. Given that the rate for short-term loans is $\frac{25}{9}$, the firm never borrows the short-term loan because if it borrows, it produces 70, while the total liability becomes $50 + \frac{250}{9}$, which is larger than 70. Thus the firm chooses not to borrow the loan, and produces 30. In this case, the firm can repay only 30 to the bank who has the claim of 50, justifying the bank's expectation. Also, the bank can repay 30 to the depositor who has the claim of 50, justifying the depositor's expectation. The bad equilibrium here is an example of a liquidity crisis, where the supply of liquidity is diminished, the short-term rate is high, and the level of output is low.

2.2 Fundamental crisis

In our framework, a fundamental crisis occurs when the level of productivity is so low that the firm necessarily goes bankrupt. Suppose now that a bad productivity shock arrives so that even with a short-term loan of 10, the firm can only produce 50, rather than 70. Without the short-term loan, the firm can only produce 30, just as before. Since the gross rate of short-term loans is greater than or equal to unity, if the firm obtains the short-term loan, its profit would be at most $50 - (50 + 10) = -10$. Thus, the firm necessarily goes bankrupt, which makes the bank default as well. Based on the same argument in the previous example, the equilibrium rates of short-term deposits and loans become $\frac{5}{3}$ and $\frac{25}{9}$, respectively. Given these rates, no short-term deposits and loans are provided, leading to a liquidity crisis where the firm's production falls to 30.

These examples illustrate how financial crises occur in our model. Notice that it captures some of the key features of a liquidity crisis: a rise in the short-term interest rates, a decline in short-term loans, and a reduction in output.

3 The model economy

Time is discrete and continues to infinity: $t = 0, 1, 2, \cdots$. Financial intermediation is introduced within the representative household framework in a similar way to Gertler and Karadi (2011).

3.1 The setup

The economy is inhabited by a unit mass of identical and infinitely-lived households. As in the standard business cycle model, each household consumes, saves, and supplies labor. In addition, in each period a firm and a bank are born in the household, and live for two periods. We abstract from capital accumulation and assume that the total supply of capital is fixed at unity.

Firms produce a single homogeneous good according to the following production technology:

$$y_t = A_t m_t^{\nu} k_t^{\alpha-\nu} l_t^{1-\alpha}, \tag{1}$$

where k_t denotes the capital input, l_t the labor input, and m_t the managerial input. Each firm supplies one unit of managerial input inelastically so that $m_t = 1$ in equilibrium. It cannot obtain the other inputs, k_t and l_t, directly from the household it belongs to. Instead, it has to purchase them at the market. Relatedly, the household cannot directly consume what its member firms produce. Thus, firms have to sell their products to other households in the market. The earnings of a firm are transferred back to the household it belongs to.

Output can also be produced via home production. Labor is the only input in the home production technology:

$$y_t = \sigma h_t, \tag{2}$$

where h_t is the labor input and $\sigma > 0$ is a constant. We shall focus on the case where σ is sufficiently small so that the home-production technology (2) is inferior to the market-production technology (1). The household can directly consume the good that is produced at home.

We assume that firms cannot obtain funds directly from the household that they belong to.[6] Firms need to borrow from banks that are members of other households.[7] Banks raise

[6]We assume that firms cannot raise equity from their own households for the sake of simplicity of exposition. This type of assumption is common in the literature. For example, see Christiano, Motto, and Rostagno 2010, Gertler and Karadi 2011, Gertler and Kiyotaki 2010.

[7]Examples of the reasons why some firms need to borrow from banks include, among others, delegated monitoring (Diamond 1984) and superior auditing technology of relationship banks (Diamond and Rajan 2000, 2001).

funds in the form of equity from the households that they belong to, and they also collect funds from other households in the form of deposits.

Regarding loans and deposits, the following three assumptions are crucial in our theory of liquidity crises. First, both loans and deposits take the form of risky debt, where borrowers make a fixed repayment as long as they are solvent.[8] Second, there are loans and deposits with different maturity. Specifically, we assume that firms need two types of loans: inter-period ("long-term") loans and intra-period ("short-term") loans. Corresponding to such financial needs of firms, banks also collect short-term and long-term deposits. Third, all loans have the same seniority regardless of their maturity. Thus the recovery rate for the short-term and long-term loans become identical. This also applies to deposits.

Let $s_t \in \Omega$ denote the state of nature in period t. We divide Ω into Ω^n and Ω^b, where Ω^n is the set of "normal" states and Ω^b is the set of "bad" states. A liquidity crisis occurs if and only if $s_t \in \Omega^b$. Note that $\Omega^n \cup \Omega^b = \Omega$ and $\Omega^n \cap \Omega^b = \emptyset$. For simplicity, we assume that s_t is i.i.d.. Let $F(s)$ denote the probability measure over Ω.

Example 1 (Sunspot Shock Economy): The first case we consider is the sunspot shock economy, in which there are no fundamental shocks but a liquidity crisis occurs as the result of self-fulfilling beliefs. In this case, $s_t \in \Omega$ denotes a sunspot shock, where $\Omega = \{n, b\}$, $\Omega^n = \{n\}$, and $\Omega^b = \{b\}$. A liquidity crisis occurs if and only if $s_t = b$. Let $\varepsilon \in [0, 1]$ denote the probability of the crisis: $F(s_t = n) = 1 - \varepsilon$, and $F(s_t = b) = \varepsilon$. The value of ε is exogenously given.

Example 2 (Fundamental Shock Economy): The second case we consider is the fundamental shock economy, where a liquidity crisis is caused by a fundamental productivity shock. In this case, s_t denotes the productivity shock: $A(s_t) = s_t$, and $\Omega = [0, +\infty)$, $\Omega^b = [0, \underline{s})$, and $\Omega^n = [\underline{s}, +\infty)$, where $\underline{s}$ is the threshold between the normal and bad states. The value of $\underline{s}$ is determined endogenously. If the productivity s_t is so small that the profit of the firm becomes negative, the firm chooses not to borrow working capital and ceases to produce the good, leading to a liquidity crisis.

3.2 Optimization problems

3.2.1 Households

The flow budget constraint for the representative household is given by

$$c_t + d_t^L + d_t^S + e_t = \tilde{\xi}_t^B R_{t-1}^D d_{t-1}^L + \tilde{\xi}_t^B R_t^B d_t^S + w_t l_t + \sigma h_t + \tilde{R}_t^E e_{t-1} + \pi_t^F, \tag{3}$$

[8] As is well known, with asymmetric information and costly state verification, the optimal contract does take the form of risky debt (e.g., Townsend 1979, Gale and Hellwig 1985).

where c_t denotes the amount of consumption, l_t the amount of labor supplied to firms (in other households), h_t the amount of labor used for home production, and π_t^F the profits earned by the member firms.

The household provides funds to banks in two different ways. First, it provides equity, e_t, to its member banks. As shown below, a moral hazard problem of banks requires them to hold some equity. The realized rate of return on equity is $\tilde{R}_t^E$. Second, each household puts deposits in banks that belong to other households. Deposits are of two types: long-term (inter-period), d_t^L, and short-term (intra-period), d_t^S. Their rates of interest are R_{t-1}^D and R_t^B, respectively.[9] Note that if $s_t \in \Omega^b$, all banks go bankrupt in period t. In such a case, the depositors recover only a fraction $\xi_t^B \in [0,1]$ of their claims. Let $\tilde{\xi}_t^B$ denote the stochastic recovery rate of depositors in period t:

$$\tilde{\xi}_t^B = \begin{cases} 1, & \text{if } s_t \in \Omega^n, \\ \xi_t^B, & \text{if } s_t \in \Omega^b. \end{cases}$$

Taking stochastic processes $(\tilde{\xi}_t^B, R_{t-1}^D, R_t^B, \tilde{R}_t^E, w_t, \pi_t^F)$ as given, the household maximizes its lifetime utility:

$$\max_{(c_t, d_t^L, d_t^S, e_t, l_t, h_t) \geq 0} E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_t + \gamma \ln(1 - l_t - h_t)], \tag{4}$$

subject to the sequence of flow budget constraints (3). The stochastic discount factor $\lambda_{t-1,t}$ is then defined as

$$\lambda_{t-1,t} = \beta \frac{c_{t-1}}{c_t},$$

and the first-order conditions for d_t^L and e_t are

$$1 = E_t \left[\lambda_{t,t+1} \tilde{\xi}_{t+1}^B R_t^D \right] = E_t \left[\lambda_{t,t+1} \tilde{R}_{t+1}^E \right]. \tag{5}$$

For a bounded solution for d_t^S to exist, $\tilde{\xi}_t^B$ and R_t^B must satisfy

$$\tilde{\xi}_t^B R_t^B \leq 1.$$

The home production technology is not used ($h_t = 0$) as long as $w_t > \sigma$.

3.2.2 Firms

Consider a firm that is born in period $t-1$. It purchases physical capital in the first year of its life, and produces output in the second. The household does not provide any funds to its member firms. Thus the firm needs to borrow from banks to purchase capital of the amount k_{t-1}. Letting q_{t-1} denote the price of capital in period $t-1$, the amount that

[9] R_{t-1}^D is the inter-period rate from $t-1$ to t, which is predetermined in period $t-1$.

the firm needs to borrow is $L_{t-1} = q_{t-1}k_{t-1}$. In period t, after s_t is realized, it decides the capital input, κ_t, and the labor input l_t. There is a rental market for capital with the rental price x_t so that the actual capital input κ_t can be different from the amount of capital purchased in the last period, k_{t-1}. The wage rate is w_t. We assume that the firm needs to pay for labor and capital services, $W_t = w_t l_t + x_t(\kappa_t - k_{t-1})$, in advance before production takes place. Thus it needs to borrow W_t from banks as working capital. This is the source of the need for firms to obtain short-term (intra-period) loans.[10] After production, the firm sells the capital at the price q_t.

The firm takes stochastic processes $(\lambda_{t-1,t}, q_{t-1}, q_t, w_t, x_t, R^L_{t-1}, R^F_t)$ as given, and chooses $(k_{t-1}, \kappa_t, l_t, W_t)$ in order to solve the profit maximization problem:[11]

$$\begin{aligned} \max_{k_{t-1}\geq 0} \quad & E_{t-1}\left[\lambda_{t-1,t}\left\{\max_{(\kappa_t,l_t,W_t)\geq 0} \pi^F_t(k_{t-1},\kappa_t,l_t,W_t)\right\}\right], \\ \text{s.t.} \quad & w_t l_t + x_t(\kappa_t - k_{t-1}) \leq W_t, \end{aligned} \tag{6}$$

where

$$\pi^F_t(k,\kappa,l,W) = \max\left\{A\kappa^{\alpha-\nu}l^{1-\alpha} + q_t k - R^L_{t-1}q_{t-1}k - R^F_t W,\ 0\right\}. \tag{7}$$

Here R^L_t is the interest rate for the inter-period loans, R^F_t is the rate for the intra-period loans, and we have used the fact that the firm chooses $m_t = 1$ in (1).

Define $r_t \equiv R^F_t x_t$. Then, using the equilibrium conditions that $k_{t-1} = \kappa_t = 1$, the first-order conditions for the firm's problem imply that:

$$\begin{aligned} & r_t = (\alpha - \nu)A_t l_t^{1-\alpha}, \\ & \int_{s\in\Omega^n} \beta\frac{c_{t-1}}{c_t}(r_t + q_t - R^L_{t-1}q_{t-1})\, dF(s) = 0. \end{aligned} \tag{8}$$

If $s_t \in \Omega^b$, all firms go bankrupt and the labor demand is zero at any wage rate. It follows that the equilibrium labor supply to firms is zero, $l_t = 0$, and households use home production to produce output, $h_t > 0$.

3.2.3 Banks

Consider a bank of a household that is born in period $t-1$. The household provides the bank with funds e_{t-1} as equity. In period $t-1$, the bank collects inter-period deposits d^L_{t-1}

[10] Here, we assume that firms cannot borrow their working capital W_t in advance in period $t-1$. This is for the sake of simplicity, and allowing it would not change the result much.

[11] Note that when $\max_{(\kappa_t,l_t,W_t)} \pi^F_t(k_{t-1},\kappa_t,l_t,W_t) = 0$ in (6), the firm is indifferent about (κ_t, l_t, W_t). To simplify the exposition, we assume that the firm chooses $\kappa_t = k_{t-1}$ and $l_t = W_t = 0$ whenever $\max_{(\kappa_t,l_t,W_t)} \pi^F_t(k_{t-1},\kappa_t,l_t,W_t) = 0$. This assumption can be easily justified by introducing a small utility cost for the firm to manage its labor input l_t.

(from other households), and makes inter-period loans L_{t-1} to firms (in other households), where $L_{t-1} = d^L_{t-1} + e_{t-1}$. In period t, it collects intra-period deposits d^S_t, and makes intra-period loans W_t to firms, so that $d^S_t = W_t$. If firms are solvent, the bank receives the scheduled amount, $R^L_{t-1}L_{t-1} + R^F_t W_t$, from them. If they go bankrupt, however, the bank can only acquire a fraction $\xi^F_t \in [0,1]$ of that amount. Let $\tilde{\xi}^F_t$ denote the recovery rate of loans to firms:

$$\tilde{\xi}^F_t = \begin{cases} 1, & \text{if } s_t \in \Omega^n, \\ \xi^F_t. & \text{if } s_t \in \Omega^b. \end{cases}$$

Then, taking into account the possibility that the bank may default, the bank's profit in period t is given by

$$\begin{aligned} &\pi^B_t(e_{t-1}, L_{t-1}, W_t) \\ &\quad = \max\Big\{\tilde{\xi}^F_t(R^L_{t-1}L_{t-1} + R^F_t W_t) - R^B_t W_t - R^D_{t-1}(L_{t-1} - e_{t-1}),\ 0\Big\}. \end{aligned} \tag{9}$$

To take into account frictions associated with financial intermediation, we assume that banks are subject to a moral hazard problem similar to the one considered by Gertler and Karadi (2011).[12] As a result, only a fraction of the bank's revenue is pledgeable to its depositors. To make the analysis simpler, we assume that the moral hazard problem is associated only with the short-term loans W_t. Specifically, suppose that the bank can divert a fraction ψ of the revenue from the short-term loans $\tilde{\xi}^F_t R^F_t W_t$, so that the pledgeable amount of the bank's revenue becomes

$$\tilde{\xi}^F_t[R^L_{t-1}L_{t-1} + (1-\psi)R^F_t W_t].$$

Thus for the bank to make short-term loans, this amount must exceed the amount of the debt that the bank owes to the depositors:

$$\tilde{\xi}^F_t[R^L_{t-1}L_{t-1} + (1-\psi)R^F_t W_t] \geq R^B_t W_t + R^D_{t-1}(L_{t-1} - e_{t-1}) \tag{10}$$

It follows that the set of feasible values of W_t, $\Gamma_t(e_{t-1}, L_{t-1})$, is defined as

$$\begin{aligned} &\Gamma_t(e_{t-1}, L_{t-1}) \\ &\quad \equiv \{0\} \cup \{W \geq 0 : \tilde{\xi}^F_t[R^L_{t-1}L_{t-1} + (1-\psi)R^F_t W] \geq R^B_t W + R^D_{t-1}(L_{t-1} - e_{t-1})\} \end{aligned} \tag{11}$$

The bank takes stochastic processes $(\lambda_{t-1,t}, e_{t-1}, \tilde{\xi}^F_t, R^D_{t-1}, R^L_{t-1}, R^B_t, R^F_t)$ as given, and chooses (L_{t-1}, W_t) to maximize the profit:

$$\max_{L_{t-1}\geq 0} E_{t-1}\left[\lambda_{t-1,t}\left\{\max_{W_t\in\Gamma_t(e_{t-1},L_{t-1})} \pi^B_t(e_{t-1}, L_{t-1}, W_t)\right\}\right], \tag{12}$$

[12] We make this assumption for a technical reason as well. Without such an assumption, the size of a bank would become infinite this model.

where the function π_t^B is defined in (9), and the correspondence Γ_t is in (11). Note that the bank provides a positive amount of short-term loans, $W_t > 0$, only if $\tilde{\xi}_t^F R_t^F \geq R_t^B$. Given the moral hazard constraint (10),

$$\tilde{\xi}_t^F (R_{t-1}^L L_{t-1} + R_t^F W_t) - R_t^B W_t - R_{t-1}^D (L_{t-1} - e_{t-1}) \geq \tilde{\xi}_t^F \psi R_t^F W_t$$

It follows that as long as it is feasible for the bank to choose $W_t > 0$, its profit is strictly positive $\pi_t^B > 0$. Thus, whenever banks default, no short-term loans are provided: $W_t = 0$, and hence all firms go bankrupt: $l_t = 0$.

It is shown later (equation 15) that π^B is linear in e_{t-1} in equilibrium. The realized return to the bank equity, $\tilde{R}_t^E$, is therefore defined by

$$\tilde{R}_t^E e_{t-1} = \pi_t^B (e_{t-1}, L_{t-1}, W_t).$$

3.3 Equilibrium

Remember that s_t is i.i.d, and there are no endogenous predetermined variables in our model economy. We thus restrict attention to equilibria where all endogenous variables are written as functions of the current state of nature $s_t \in \Omega$. Banks and firms go bankrupt in period t if and only if $s_t \in \Omega^b$. In particular, for $s_t \in \Omega^n$, $W(s_t) > 0$, $l(s_t) > 0$, $h(s_t) = 0$, and for $s_t \in \Omega^b$, $W(s_t) = l(s_t) = 0$ and $h(s_t) > 0$.

The equilibrium conditions for the capital stock, managerial inputs, loans, deposits, and consumption are given by

$$\begin{aligned} k(s) = \kappa(s) = m(s) = 1, &\quad \text{for all } s \in \Omega, \\ L(s) = q(s)k(s) = d^L(s) + e(s), &\quad \text{for all } s \in \Omega, \\ W(s) = w(s)l(s) = d^S(s), &\quad \text{for all } s \in \Omega, \\ c(s) = \begin{cases} A(s)l(s)^{1-\alpha}, & \text{for } s \in \Omega^n, \\ \sigma h(s), & \text{for } s \in \Omega^b. \end{cases} \end{aligned}$$

Consider a bank born in period $t-1$. Given the limited liability (9), it only cares about its profits in normal states $s_t \in \Omega^n$. Thus, if $R^L(s_{t-1}) > R^D(s_{t-1})$ then it would choose $L(s_{t-1}) = \infty$, and if $R^L(s_{t-1}) < R^D(s_{t-1})$ then $L(s_{t-1}) = 0$. In either case, the equilibrium condition $L(s_{t-1}) = q(s_{t-1}) > 0$ would be violated. As a result,

$$R^L(s) = R^D(s) \equiv R(s).$$

In equilibrium, the short-term (i.e., intra-period) rate on bank deposits, $R^B(s_t)$, is equal to unity if $s_t \in \Omega^n$:

$$R^B(s) = \frac{1}{\tilde{\xi}^B(s)} = \begin{cases} 1, & \text{for } s \in \Omega^n, \\ \frac{1}{\xi^B(s)}, & \text{for } s \in \Omega^b, \end{cases}$$

The interest rate on short-term loans to firms, $R^F(s)$, during a crisis is

$$R^F(s) = R^F_b = \frac{1}{\xi^F \xi^B}, \qquad s \in \Omega^b. \tag{13}$$

Unlike $R^B(s)$, $R^F(s)$ may be greater than unity in normal states $s \in \Omega^n$ because of the moral hazard problem of banks ($R^F(s) = 1$ for $s \in \Omega^n$ if (10) does not bind).

Given $R^B(s)$, the enforcement constraint (10) of the bank for $s_t \in \Omega^n$ becomes

$$W(s_t) \leq \frac{R(s_{t-1})e(s_{t-1})}{1-(1-\psi)R^F(s_t)}, \quad \text{for } s_{t-1} \in \Omega \text{ and } s_t \in \Omega^n.$$

We restrict attention to the case where this constraint binds for $s_t \in \Omega^n$:

$$W(s') = w(s')l(s') = \frac{R(s)e(s)}{1-(1-\psi)R^F(s')}, \quad \text{for } s \in \Omega \text{ and } s' \in \Omega^n. \tag{14}$$

It follows that $R(s)e(s) = [1-(1-\psi)R^F(s')]W(s')$ is a constant which does not depend on s or s'.

Given that $\tilde{\xi}^F(s_t) = 1$ for $s_t \in \Omega^n$, and $R^L(s_{t-1}) = R^D(s_{t-1}) = R(s_{t-1})$, the realized profit of the bank is

$$\begin{aligned} \pi^B_t &= \begin{cases} R^F(s_t)W(s_t) - W(s_t) + R(s_{t-1})e(s_{t-1}), & \text{for } s_t \in \Omega^n, \\ 0, & \text{for } s_t \in \Omega^b, \end{cases} \\ &= \begin{cases} \frac{\psi R^F(s_t)}{1-(1-\psi)R^F(s_t)} R(s_{t-1})e(s_{t-1}), & \text{for } s_t \in \Omega^n, \\ 0, & \text{for } s_t \in \Omega^b. \end{cases} \end{aligned} \tag{15}$$

The expected profit of the bank can then be written as

$$E_{t-1}[\lambda_{t-1,t}\pi^B_t] = \left\{ \int_{s_t \in \Omega^n} \beta \frac{c(s_{t-1})}{c(s_t)} \frac{\psi R^F(s_t)}{1-(1-\psi)R^F(s_t)}\, dF(s_t) \right\} R(s_{t-1})e(s_{t-1}).$$

It follows that

$$E_{t-1}[\lambda_{t-1,t}\tilde{R}^E_t] = \left\{ \int_{s_t \in \Omega^n} \beta \frac{c(s_{t-1})}{c(s_t)} \frac{\psi R^F(s_t)}{1-(1-\psi)R^F(s_t)}\, dF(s_t) \right\} R(s_{t-1}) \tag{16}$$

The first-order conditions (5) of the household's utility maximization problem imply that the expected returns on bank equity and bank deposit should be equal in equilibrium:

$$E_{t-1}[\lambda_{t-1,t}\tilde{\xi}^B_t R^D_{t-1}] = E_{t-1}[\lambda_{t-1,t}\tilde{R}^E_t] = 1. \tag{17}$$

The left-hand side of this equation is

$$E_{t-1}[\lambda_{t-1,t}\tilde{\xi}^B_t R^D_{t-1}] = \beta \left\{ \int_{s_t \in \Omega^n} \frac{c(s_{t-1})}{c(s_t)}\, dF(s_t) + \int_{s_t \in \Omega^b} \frac{c(s_{t-1})}{c(s_t)} \xi^B(s_t)\, dF(s_t) \right\} R(s_{t-1}). \tag{18}$$

It follows from (16) and (18) that the first equation in (17) can be rewritten as

$$\int_{s \in \Omega^n} \frac{1}{c(s)} \frac{\psi R^F(s)}{1-(1-\psi)R^F(s)}\, dF(s) = \int_{s \in \Omega^n} \frac{1}{c(s)}\, dF(s) + \int_{s \in \Omega^b} \frac{1}{c(s)} \xi^B(s)\, dF(s). \tag{19}$$

Note that if the probability of a liquidity crisis is zero, i.e., $F(\Omega^b) = 0$, then $R^F(s) = 1$ for $s \in \Omega^n$, just like $R^B(s)$. Otherwise, however, $R^F(s) > 1$ for $s \in \Omega^n$. This is because during a crisis, bank deposits pay a fraction ξ^B, but the value of the equity of the bank reduces to zero. To compensate this difference, the bank equity needs to yield a higher return in normal states $s \in \Omega^n$ by making $R^F(s) > 1$.

The second equation in (17) becomes

$$\beta \left\{ \int_{s' \in \Omega^n} \frac{1}{c(s')} \, dF(s') + \int_{s' \in \Omega^b} \frac{1}{c(s')} \xi^B(s') \, dF(s') \right\} c(s)R(s) = 1, \qquad s \in \Omega. \tag{20}$$

Thus, $c(s)R(s)$ is a constant that does not depend on $s \in \Omega$.

The first-order condition (8) of firms is rewritten as

$$\int_{s' \in \Omega^n} \frac{1}{c(s')} \, dF(s') R(s) q(s) = \int_{s' \in \Omega^n} \frac{1}{c(s')} \{r(s') + q(s')\} \, dF(s'), \qquad s \in \Omega. \tag{21}$$

Note that $R(s)q(s)$ does not depend on $s \in \Omega$.

When $s \in \Omega^b$, output is produced only through home production, and therefore,

$$c(s) = c_b \equiv \frac{\sigma}{1+\gamma}, \qquad s \in \Omega^b, \tag{22}$$

$$h(s) = h \equiv \frac{1}{1+\gamma}, \qquad s \in \Omega^b, \tag{23}$$

$$l(s) = 0, \qquad s \in \Omega^b, \tag{24}$$

$$w(s) = w_b \equiv \sigma, \qquad s \in \Omega^b, \tag{25}$$

$$r(s) = 0, \qquad s \in \Omega^b. \tag{26}$$

It follows from (20) and (21) that when $s \in \Omega^b$ we can write as $q(s) = q_b$ and $R(s) = R_b$.

When $s \in \Omega^n$, utility maximization of households, profit maximization of firms, and market clearing imply

$$c(s) = A(s)l(s)^{1-\alpha}, \qquad s \in \Omega^n, \tag{27}$$

$$w(s) = \frac{\gamma A(s) l(s)^{1-\alpha}}{1 - l(s)}, \qquad s \in \Omega^n, \tag{28}$$

$$R^F(s) w(s) = (1-\alpha) A(s) l(s)^{-\alpha}, \qquad s \in \Omega^n, \tag{29}$$

$$r(s) = (\alpha - \nu) A(s) l(s)^{1-\alpha}, \qquad s \in \Omega^n, \tag{30}$$

$$h(s) = 0, \qquad s \in \Omega^n. \tag{31}$$

When $s_t \in \Omega^n$, both banks and firms are solvent so that the equilibrium recovery rates of their debt are unity:

$$\xi^F(s) = 1, \qquad \text{for } s \in \Omega^n,$$

$$\xi^B(s) = 1, \qquad \text{for } s \in \Omega^n.$$

When $s_t \in \Omega^b$, both banks and firms go bankrupt, and they do not obtain short-term loans/deposits: $W(s_t) = 0$, and thus $l(s_t) = 0$. Thus the firm's revenue in such a state is $q(s_t)k(s_{t-1}) = q_b$. The value of the debt is $R^L(s_{t-1})L(s_{t-1}) = R(s_{t-1})q(s_{t-1})$. It follows that the recovery rate of loans to defaulting firms, ξ^F, is determined by

$$q_b - \xi^F R(s_{t-1})q(s_{t-1}) = 0.$$

Here, notice that $R(s)q(s)$ is independent of $s \in \Omega$, as shown in (21), and, in particular, $R(s)q(s) = R_b q_b$. It follows that

$$\xi^F = \frac{q_b \int_{s\in\Omega^n} c(s)^{-1}\, dF(s)}{\int_{s\in\Omega^n} c(s)^{-1}\{r(s) + q(s)\}\, dF(s)} = \frac{1}{R_b}. \tag{32}$$

Similarly, the recovery rate of bank deposits in the crisis, ξ^B, is determined by

$$\xi^F R(s_{t-1})L(s_{t-1}) - \xi^B R(s_{t-1})\big[L(s_{t-1}) - e(s_{t-1})\big],$$

which is rewritten as

$$\xi^B = \xi^F \frac{R(s)q(s)}{R(s)q(s) - R(s)e(s)} > \xi^F. \tag{33}$$

Here, again, note that $R(s)q(s)$ and $R(s)e(s)$ are independent of s. The model parameters are restricted so that the value of ξ^B defined in (33) is less than one (and hence $\xi^F < 1$).

When we consider the sunspot shock economy, $F(s = b) = \varepsilon$ is exogenously given. On the other hand, when we consider the fundamental shock economy, the threshold value $\underline{s}$ is determined endogenously by the break-even condition:

$$\begin{aligned} &A(\underline{s})l(\underline{s})^{1-\alpha} - R^F(\underline{s})w(\underline{s})l(\underline{s}) + q(\underline{s}) \\ &\quad - \frac{1}{\int_{\underline{s}}^{\infty} c(s)^{-1}\, dF(s)} \int_{\underline{s}}^{\infty} c(s)^{-1}\{r(s) + q(s)\}\, dF(s) = 0. \end{aligned} \tag{34}$$

If s_t is below this threshold, all firms go bankrupt and output is exclusively produced via home production.

A competitive equilibrium is given by a collection of functions $\{c(s),\ l(s),\ h(s),\ w(s),\ e(s),\ r(s),\ q(s),\ R(s),\ R^F(s),\ \xi^F,\ \xi^B,\ \underline{s}\}$ that satisfies (13), (14), and (19)-(34). They must be non-negative. (For the sunspot economy, remove $\underline{s}$ from the definition of the equilibrium.)

4 Equilibrium in the sunspot shock economy

Here we consider the sunspot shock economy, where there are two states, n and b, and $\Omega = \{n, b\}$, $\Omega^n = \{n\}$, $\Omega^b = \{b\}$, $\Pr(s_t = n) = 1 - \varepsilon$, and $\Pr(s_t = b) = \varepsilon$. The level of productivity is constant: $A(s) = A$ for all $s \in \Omega$. Since the state of the economy is a sunspot variable, the bad outcome is caused purely by self-fulfilling beliefs. The value of each variable in states n and b are denoted by subscripts n and b, respectively. The set of equilibrium conditions for the sunspot economy are given in Appendix.

When $s_t = n$, firms and banks born in period $t-1$ are solvent in period t. These firms obtain short-term loans $W_n = w_n l_n$, hire labor $l_t = l_n$, use capital $\kappa_t = k_{t-1} = 1$, and produce output $y_t = Al_n^{1-\alpha} = c_n$. Because of the borrowing constraint, the level of output is lower than the first-best level (see equations (6)-(7)). The profit of these firms is

$$\pi_t^F = \pi_n^F \equiv Al_n^{1-\alpha} + q_n - R(s_-)q(s_-) - R_n^F W_n > 0,$$

wheres_- is the state in the previous period, and $R_n q_n = R_b q_b$ as shown in equation (21). The firms' revenue consists of sales of output and capital: $Al_t^{1-\alpha} + q_t$. The cost $R_{t-1}q_{t-1} + R_t^F W_t$ represents the repayment of the inter-period loan q_{t-1} and the intra-period loan W_t. The former is used to purchase capital, $k_{t-1} = 1$, in period $t-1$ and the latter is to pay for the wage bill, $W_t = w_t l_t$, in period t.

The firms obtain those funds from the banks. The banks' profit in the normal state is

$$\pi_t^B = \pi_n^B \equiv R(s_-)q(s_-) + R_n^F W_n - R_n^B W_n - R(s_-)(q(s_-) - e(s_-)) > 0,$$

where $R_n e_n = R_b e_b$ as shown in equation (14). Here, $R(s_-)q(s_-) + R_n^F W_n$ is the repayment from the firms and $R_n^B W_n + R(s_-)(q(s_-) - e(s_-))$ the payment to the depositors.

Notice that in period t, firms and banks are indebted with inter-period loans, $R_{t-1}q_{t-1}$ and $R_{t-1}(q_{t-1} - e_{t-1})$, respectively, which constitute their fixed costs. In the normal state, banks and firms earn positive profits because the short-term rates, R_t^F and R_t^B, are sufficiently low. Otherwise, they could go default, and a crisis occurs.

In the sunspot economy, a crisis occurs due to self-fulfilling beliefs. Suppose that when $s_t = b$, everyone believes that $R_t^F = R_b^F \equiv 1/(\xi^F \xi^B)$ and $R_t^B = R_b^B \equiv 1/\xi^B$, where $\xi^F < \xi^B < 1$ are defined in (32) and (33), respectively. Later we verify that these beliefs are rational. We assume parameter values such that

$$\max_{l \geq 0} \Big\{ Al^{1-\alpha} + q_b - R(s_-)q(s_-) - R_b^F w_b l \Big\} < 0, \tag{35}$$

$$\xi^F R(s_-)q(s_-) - R(s_-)\big[q(s_-) - e(s_-)\big] < 0, \tag{36}$$

where s_- denotes the state in the previous period, and $w_b = \sigma$. These assumption guarantee that firms and banks go bankrupt when $s_t = b$: Condition (35) implies that, given the short-term loan rate R_b^F and inter-temporal debt $R(s_-)q(s_-)$, it is impossible for firms to make a positive profit. When firms go bankrupt, banks can recover only a fraction ξ^F of their inter-temporal loans $R(s_-)q(s_-)$. Condition (36), then, guarantees that banks also become insolvent when $s_t = b$.

With $R_t^F = R_b^F$ and $R_t^B = R_b^B$, firms' demand for intra-temporal loans is zero; banks and households are indifferent about the amount of intra-temporal loans and deposits.

With $w_t = w_b$, firms' demand for labor is zero, and households are indifferent between supplying l and h. Thus, markets are cleared with theres prices.

Our sunspot economy exhibits equilibrium fluctuations in the following fashion. In the normal state, which occurs with probability $1 - \varepsilon$, the short term interest rates are low: $R_n^B = 1$ and $R_n^F \approx 1$, short-term funds $W_n = w_n l_n$ flow from households to banks, and from banks to firms, and the level of economic activity is high, $y_n = A l_n^{1-\alpha}$. A liquidity crisis occurs with probability ε, where short-term rates rise to R_b^F and R_b^B, the supply of short-term liquidity is evaporated, $W_b = 0$, and market activity vanishes, $y_b = l_b = 0$.

Figure 1 illustrates numerically what happens during a liquidity crisis in the sunspot shock economy. The parameter values are chosen so that $\varepsilon = 0.01$, $\beta = 0.96$, $\nu = 0.1$, $\alpha = 0.4$, $A = 1$, $\gamma = 1.4$, $\sigma = 0.8741$, and $\psi = 0.1$. In particular, a crisis occurs with probability 0.01 in each period. The figure plots the time paths of output (c), interest rates (R, R^F, R^B), short-term loans (W), profits (π^F, π^B), and the price of capital (q) in the case where a liquidity crisis occurs in period 0, that is, $s_t = n$ for $t \neq 0$ and $s_0 = b$.

5 Equilibrium in the fundamental shock economy

In this section we consider productivity shocks and abstract from sunspot shocks. The state of nature $s_t \in \Omega = [0, +\infty)$ denotes the productivity level in period t, $A(s_t) = s_t$. The productivity shock s_t is i.i.d. across periods with probability distribution function $F(s_t)$. The state space Ω is divided into $\Omega^b = [0, \underline{s})$ and $\Omega^n = [\underline{s}, +\infty)$, where the threshold value, $\underline{s}$, is determined endogenously. We continue to restrict our attention to the case where net worth constraint (14) binds as long as firms are solvent.[13]

In the sunspot economy, the fundamental parameters of the economy are fixed. Nev-

[13] Let $\overline{s}$ denote the threshold value such that for $s \leq \overline{s}$ the net worth constraint (14) does not bind. We assume parameter values so that $\overline{s} < \underline{s}$. If, however, $\overline{s} > \underline{s}$, then the equilibrium values for $s \in [\underline{s}, \overline{s}]$ are computed as follows. The value of $\overline{s}$ is determined as the solution to

$$w^*(\overline{s}) l^*(\overline{s}) = \frac{R_{t-1} e_{t-1}}{\psi},$$

where $l^*(s)$ and $w^*(s)$ are determined by

$$\frac{\gamma A(s) l(s)^{1-\alpha}}{1 - l(s)} = (1-\alpha) A(s) l(s)^{-\alpha},$$

$$w^*(s) = \frac{\gamma A(s) l(s)^{1-\alpha}}{1 - l(s)}.$$

Then for $s \in [\underline{s}, \overline{s}]$ we have

$$R^F(s) = 1,$$
$$l(s) = l^*(s),$$
$$w(s) = w^*(s).$$

ertheless, there are two sets of prices that are consistent with market clearing in each period. One of them allows firms and banks to earn positive profits, but the other makes them insolvent and leads to a liquidity crisis. Which set of prices realize is determined by self-fulfilling beliefs.

Here, we abstract from this type of crisis. Instead, we restrict attention to the class of equilibria where a crisis occurs only when the productivity is too low for firms and banks to earn positive profits. When $s \in \Omega^n = [\underline{s}, +\infty)$, firms earn positive profits:

$$\pi^F(s) = A(s)l(s)^{1-\alpha} + q(s) - R(s_-)q(s_-) - R^F(s)w(s)l(s) \geq 0.$$

The threshold value, $\underline{s}$, is determined by $\pi^F(\underline{s}) = 0$.

When the productivity level falls below the threshold level, $s < \underline{s}$, a liquidity crisis occurs. What happens during the crisis is similar to that in the sunspot economy: Firms and banks go bankrupt; the supply of short-term loans evaporates, $W(s) = W_b = 0$; the short-term interest rates rise sharply, $R^F(s) = R_b^F \equiv 1/(\xi^F \xi^B)$ and $R^B(s) = R_b^B \equiv 1/\xi^B$; the level of output declines, $y(s) = y_b \equiv \sigma h$. Details of calculation of the equilibrium in the fundamental shock economy are given in Appendix.

Now consider a numerical example, where the parameter values are set as $\beta = 0.95$, $\nu = 0.097$, $\alpha = 0.3$, $\sigma = 0.87$, and $\psi = 0.1$. The productivity shock $\ln(s)$ is assumed to follow a normal distribution with mean 0 and standard deviation 0.01. Then, as shown in Table 1, $\underline{s} = 0.98$, and $F(\underline{s}) = 0.0094$. Thus, on average, a liquidity crisis occurs about once in one hundred years. The equilibrium dynamics is illustrated in Figure 2, where the horizontal axis for each panel is the time index. The productivity level, $A_t = s_t$, is realized as shown in the top-left panel. Here, s_t is greater than $\underline{s}$ except for $t = 0$. Thus a liquidity crisis occurs (only) in period 0. The figure illustrates the key features of the crisis discussed above (high interest rates and low loans and output). In addition, notice that a liquidity crisis works as a magnifying mechanism of productivity shocks. Indeed, the productivity level declines only slightly from period -1 to period 0. Nevertheless, such a small decline in the productivity level results in a huge reduction in the economic activity.

6 Policy analysis

So far we have restricted our attention to the case without government intervention. Probably a most typical form of government intervention during a financial crisis is to subsidize banks in some way. As an example of such a policy, we examine the effects of a policy that guarantees bank deposits in this section. Specifically, we suppose that

- the government gives subsidy to banks if and only if $s \in \Omega^b$. The amount of the subsidy is determined in such a way that in equilibrium $\tilde{\xi}^B(s) = 1$ for both $s \in \Omega^n$ and $s \in \Omega^b$, and the return on the bank equity is zero when $s \in \Omega^b$;

- firms do not receive any subsidy from the government; and
- the fund for the subsidy is raised by lump-sum taxes on households.

Note that the government does not save firms nor holders of the bank equity. Here, it only saves depositors.

Remember that the conditions for the laissez-faire equilibrium are given by (13), (14), and (19)-(34). When deposits are guaranteed, (19), (20), and (33) among these conditions should be replaced by the following ones:

$$\int_{s\in\Omega^n} \frac{1}{c(s)} \frac{\psi R^F(s)}{1-(1-\psi)R^F(s)}\, dF(s) = \int_{s\in\Omega} \frac{1}{c(s)}\, dF(s),$$

$$\beta \left\{ \int_{s'\in\Omega} \frac{1}{c(s')}\, dF(s') \right\} c(s)R(s) = 1, \qquad s\in\Omega,$$

$$\xi^B(s) = 1, \qquad s\in\Omega.$$

6.1 Policy intervention in the sunspot shock economy

Let us start with examining how guaranteeing bank deposits would affect the likelihood of the sunspot crisis. Here we say "the likelihood of the sunspot crisis is high (low)" when the region of parameter values in which the sunspot crisis exists is large (small). See Appendix for the equilibrium conditions with deposit guarantee.

Figures 3 and 4 illustrate its effect for the benchmark parameter values: $\beta = 0.96$, $\nu = 0.1$, $\alpha = 0.4$, $A = 1$, $\gamma = 1.4$, $\sigma = 0.8741$, and $\psi = 0.1$. Figure 3 plots the upper bound of the probability of a crisis, $\bar{\varepsilon}$, for different values of the productivity of home production, σ, where all the other parameter values are fixed at the benchmark values. That is, for each σ, there is a sunspot equilibrium for any $\varepsilon \in [0, \bar{\varepsilon}(\sigma)]$. The solid line in the figure represents $\bar{\varepsilon}(\sigma)$ under the laissez-faire policy, and the dashed line shows it under the deposit guarantee policy. Figure 4 plots $\bar{\varepsilon}$ under the two policies as a function of ψ, i.e., the strength of moral hazard of banks.

In both figures, it is apparent that the upper bound of the probability of a crisis is much smaller under the deposit guarantee policy than under the laissez-faire policy. That is, by guaranteeing bank deposits, the government can significantly reduce the likelihood of the sunspot crisis. Under the laissez-faire policy, if a crisis occurs, the short-term rates would rise to $R_b^B = 1/\xi^B$, and $R_b^F = 1/(\xi^B\xi^F)$. On the other hand, if bank deposits are guaranteed, a crisis would not affect the short-term rate on bank deposits, i.e., $R_b^B = 1$. As a result, the interest rate on short-term loans during a crisis would be $R_b^F = 1/\xi^F$. Thus, R_b^F is smaller under the deposit guarantee policy than under the laissez-faire policy, and this is why guaranteeing bank deposits reduces the likelihood of sunspot crises.

It is worth noting, however, that the deposit guarantee policy does not entirely eliminate it. This is in contrast with the bank run model such as Diamond and Dybvig (1983), where guaranteeing bank deposits does rule out the possibility of sunspot crises. The difference between the two models is due to the fact that our model takes into account the possibility of default of firms as well as that of banks. Even though the deposit guarantee policy reduces R_b^F, it is still possible that expecting $R^F = R_b^F$ causes firms to go bankrupt, driving a sunspot crisis.

6.2 Policy intervention in the fundamental shock economy

Next consider how fundamental crisis is affected by the deposit guarantee policy. (The equilibrium conditions are given in Appendix.) Table 1 shows the value of $\underline{s}$ both under the laissez-faire policy and under the deposit guarantee policy for the parameter values given in Section 5. Guaranteeing bank deposits increases $\underline{s}$ from 0.9768 to 0.9796, and the probability of fundamental crisis rises from about one percent to 2 percent. Thus, such policy doubles the likelihood of fundamental crisis from about once in one hundred years to about once in fifty years. Therefore, the economy becomes more susceptible to financial crises if the government commits to bailout banks.

There is simple intuition for this result. If the government is expected to guarantee bank deposits when $s \in \Omega^b$, the expected return to bank deposits goes up because $\xi^B(s) = 1$ even when $s \in \Omega^b$. The higher return on deposits tends to reduce the supply of bank equity, which tightens the moral hazard constraint (62), and increases the short-term interest rate on corporate loans, $R^F(s)$. Higher $R^F(s)$, in turn, squeezes the profit of firms, leading to an increase in the threshold value of productivity $\underline{s}$.

This result is related to "over-leverage" induced by the government's bailout commitment (Bianchi 2012, Keister 2012). The deposit guarantee induces banks to pursue higher leverage, which reduces the profits of firms and increases the risk of financial crisis. So, as far as the fundamental crisis is concerned, the government's commitment to guarantee bank deposits makes the social welfare strictly worse.

The results in this section illustrates the importance of distinguishing the type of crisis in order to design effective policy interventions. Taking the deposit guarantee policy as an example, it is shown that such policy is effective in reducing the likelihood of sunspot crises, but has a side effect of increasing the probability of a fundamental crisis.

7 Conclusion

We have proposed a new simple mechanism of how systemic financial crises occur. It is based on debt overhang in short-term loans. A crisis can be caused either by self-fulfilling beliefs or by a fundamental shock to the economy. During the crisis, the supply of short-term loans drops sharply, the short-term interest rate rises, and production activities are depressed. Our model roughly captures some of the key features observed during actual financial crises.

We also have examined the effects of guaranteeing bank deposits during a crisis. Such policy has the following type of trade-offs. On the one hand, it reduces the possibility of self-fulfilling crises. On the other hand, however, it raises the probability of fundamental crises. The overall welfare effect of such policy would depend on the probabilities of sunspot and fundamental crises. Ideally, policy intervention should be contingent on the type of crisis.

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Table 1: Threshold value $\underline{s}$

	$\underline{s}$	$F(\underline{s})$
(1) laissez faire	0.9768	0.94%
(2) bank bailout	0.9796	1.95%

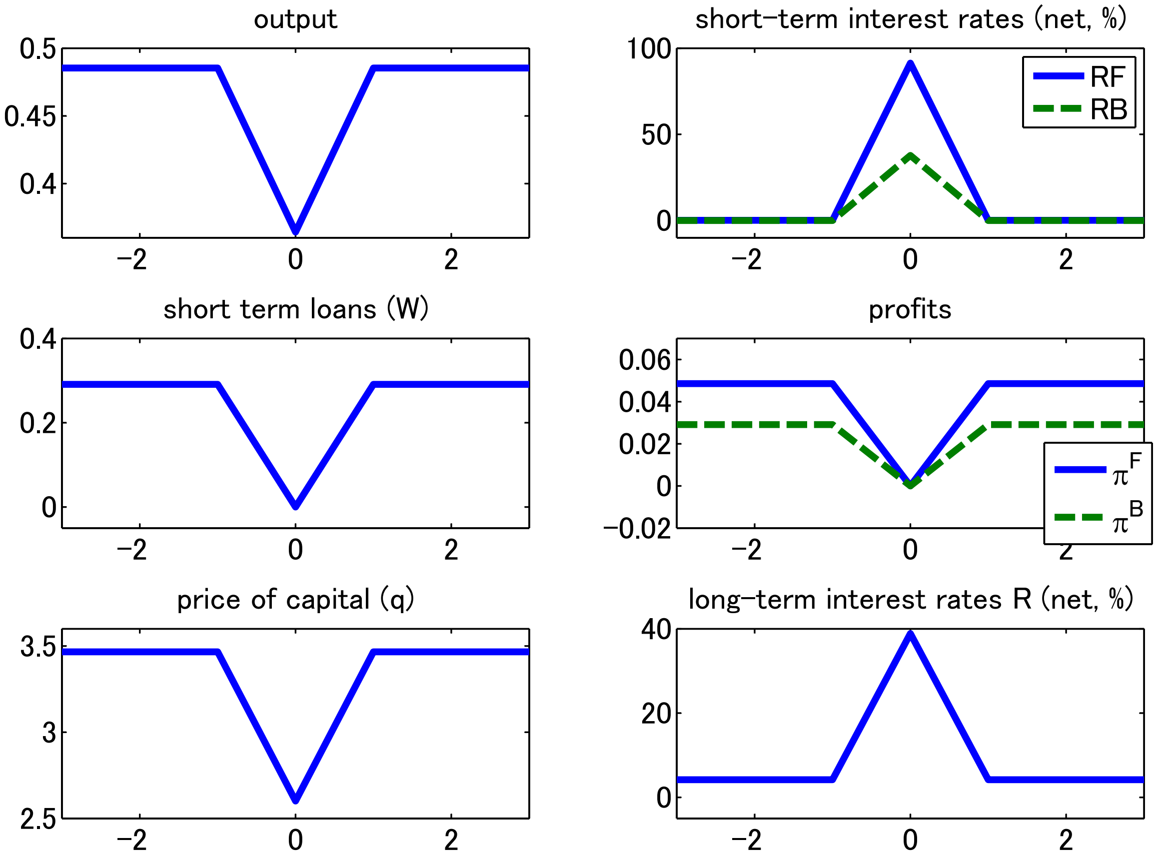

Figure 1: Liquidity crisis in the sunspot shock economy. The horizontal axis in each panel is the time period. A crisis occurs in period 0.

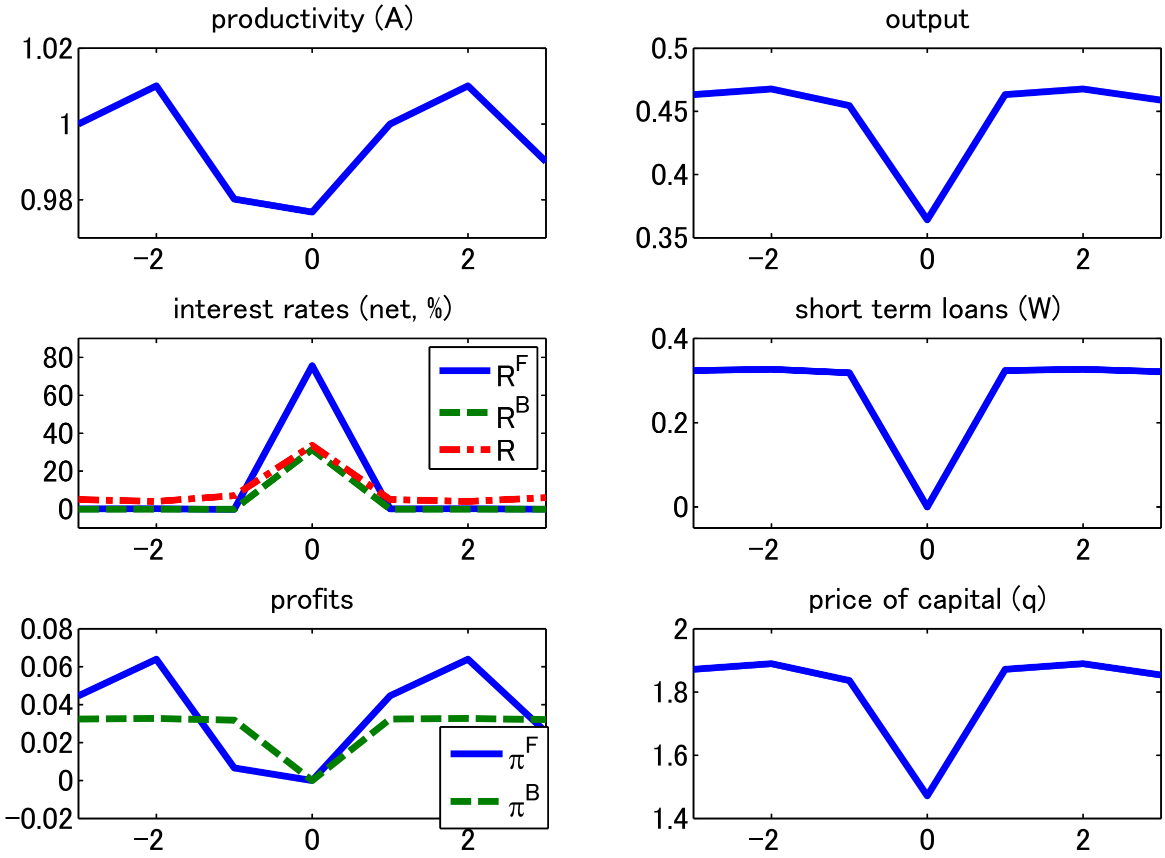

Figure 2: Liquidity crisis in the fundamental shock economy. The horizontal axis in each panel is the time period. A crisis occurs in period 0.

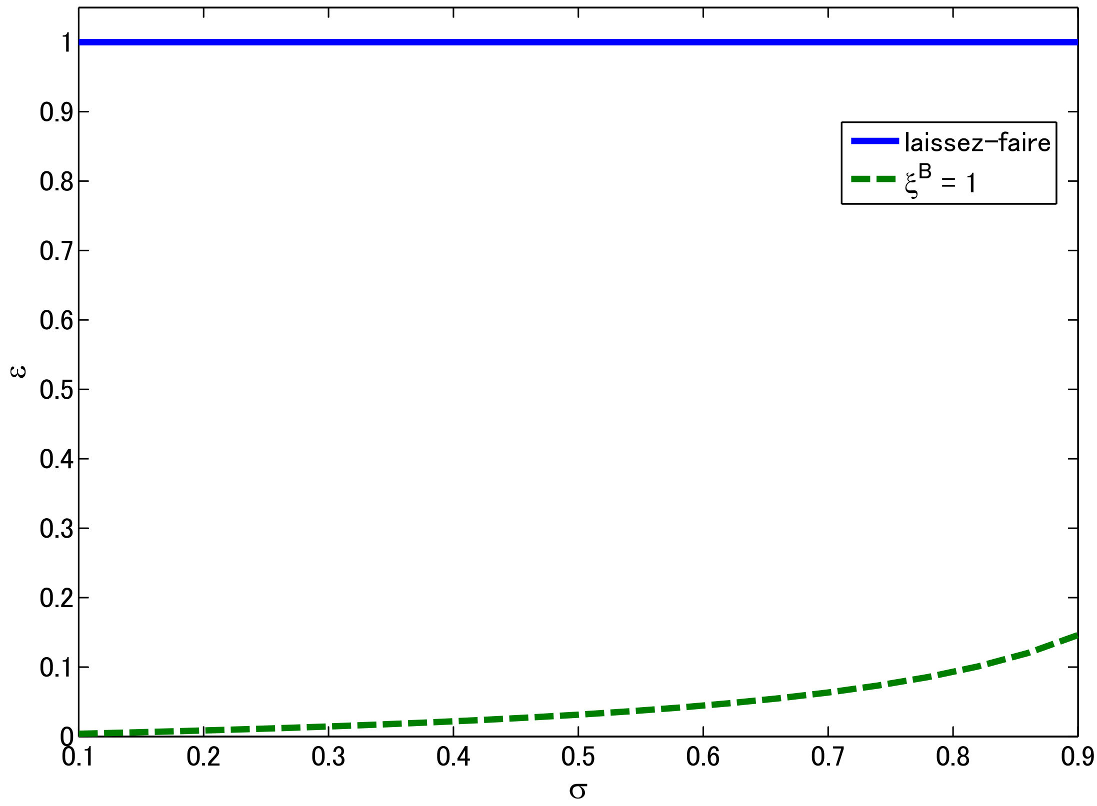

Figure 3: The upper bound of the probability of a crisis ($\bar{\varepsilon}$) in the class of sunspot equilibria for different values of the productivity of home production (σ). The solid line denotes the case without government intervention. The dashed line corresponds to the case where the government guarantees bank deposits so that $\xi^B = 1$.

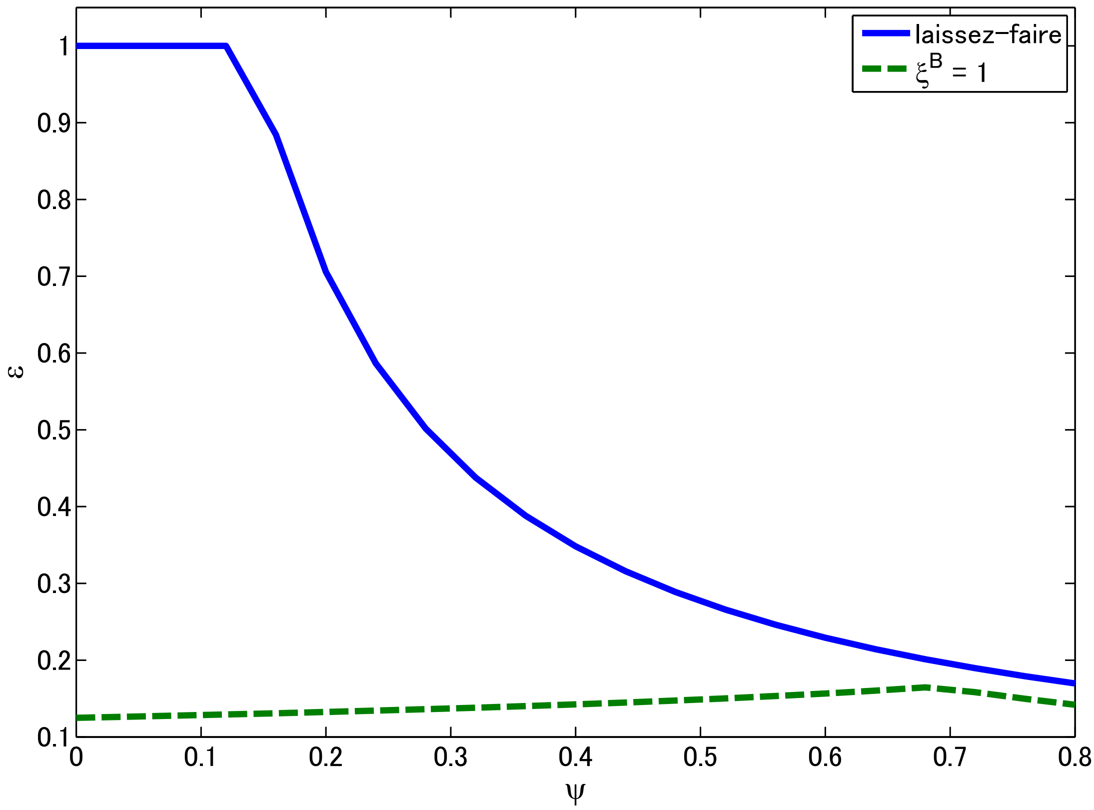

Figure 4: The upper bound of the probability of a crisis ($\bar{\varepsilon}$) in the class of sunspot equilibria for different values of the degree of moral hazard of banks (ψ). The solid line denotes the case without government intervention. The dashed line corresponds to the case where the government guarantees bank deposits so that $\xi^B = 1$.

A Appendix

A.1 The equilibrium conditions for the sunspot economy

Consider the profit maximization problem (6) of a firm born in period $t-1$. Given the state in that period, s_{t-1}, the first-order condition with respect to k leads to

$$r_n + q_n = R(s_{t-1})q(s_{t-1}).$$

Equation (19) implies that

$$(1-\varepsilon)\frac{\psi R_n^F}{1-(1-\psi)R_n^F} = 1-\varepsilon+\varepsilon\xi^B\frac{c_n}{c_b},$$

which is rewritten as

$$R_n^F = 1 + \frac{\psi\varepsilon\xi^B\frac{c_n}{c_b}}{1-\varepsilon+(1-\psi)\varepsilon\xi^B\frac{c_n}{c_b}} \tag{37}$$

The household's utility maximization problem (4) implies that when $s_t = b$

$$h_b = \frac{1}{1+\gamma}, \tag{38}$$

$$c_b = \frac{\sigma}{1+\gamma}. \tag{39}$$

The firm's and bank's profit maximization and the household's utility maximization imply

$$\frac{R_n^F\gamma A l_n^{1-\alpha}}{1-l_n} = (1-\alpha)Al_n^{-\alpha}, \tag{40}$$

$$c_n = Al_n^{1-\alpha}, \tag{41}$$

$$r_n = (\alpha-\nu)Al_n^{1-\alpha}, \tag{42}$$

$$R_n = \left[\beta\left\{1-\varepsilon+\varepsilon\frac{c_n}{c_b}\xi^B\right\}\right]^{-1}, \tag{43}$$

$$R_b = \frac{c_n}{c_b}R_n, \tag{44}$$

$$q_n = \frac{1}{R_n}[r_n+q_n], \tag{45}$$

$$q_b = \frac{1}{R_b}[r_n+q_n], \tag{46}$$

$$w_n = \frac{\gamma A l_n^{1-\alpha}}{1-l_n}, \tag{47}$$

$$W_n = w_n l_n, \tag{48}$$

$$W_n = \frac{R(s_{t-1})e(s_{t-1})}{1-(1-\psi)R_n^F} = \frac{R_n e_n}{1-(1-\psi)R_n^F} = \frac{R_b e_b}{1-(1-\psi)R_n^F}. \tag{49}$$

By definition, the recovery rates during a crisis, ξ^F and ξ^B, are given by

$$\xi^F = \frac{1}{R_b}, \tag{50}$$

$$\xi^B = \frac{\xi^F[R(s_{t-1})L(s_{t-1})+R_b^F W_b]}{R_b^B W_b + R(s_{t-1})[L(s_{t-1})-e(s_{t-1})]} = \frac{q_b}{r_n+q_n-[1-(1-\psi)R_n^F]w_n l_n}, \tag{51}$$

where we have used the fact that $L(s_{t-1}) = q(s_{t-1})k(s_{t-1}) = q(s_{t-1})$, $\xi^F = q_b/[R(s_{t-1})q(s_{t-1})]$, $R(s_{t-1})q(s_{t-1}) = R_n q_n = R_b q_b = r_n + q_n$, and $W_b = 0$.

The solution to the system of equations (37)–(51) provides a candidate of sunspot equilibrium where a liquidity crisis occurs with probability ε. It is indeed an equilibrium only if it satisfies some consistency conditions. First, it must be the case that firms default when $s_t = b$. The condition for this is given as follows. Define $\underline{R}^F$ by

$$\xi^F \underline{R}^F = R_b^B = \frac{1}{\xi^B},$$

that is, $\underline{R}^F = 1/(\xi^F \xi^B)$. Banks would supply a positive amount of working capital loans at $s_t = b$, $W_b > 0$, only if $R_b^F \geq \underline{R}^F$. Now define $\bar{R}^F$ as the maximum rate of interest that is consistent with $l_b > 0$. Given that the wage rate is σ at $s_t = b$, if $l_b > 0$, it must satisfy $(1-\alpha)Al_b^{-\alpha} = \bar{R}^F \sigma$. When $R_b^F = \bar{R}^F$, the firm's profit at $s_t = b$ is just equal to zero so that $\alpha A l_b^{1-\alpha} = r_n + q_n - q_b$. It follows that

$$\bar{R}^F = \frac{(1-\alpha)A}{\sigma} \left(\frac{\alpha A}{r_n + q_n - q_b} \right)^{\frac{\alpha}{1-\alpha}}$$

If $\bar{R}^F < \underline{R}^F$, it is indeed the case that $l_b = W_b = 0$ and firms default at $s_t = b$. The condition $\bar{R}^F < \underline{R}^F$ is rewritten as:

$$\frac{(\alpha A)^{\frac{\alpha}{1-\alpha}}(1-\alpha)A}{\left[\frac{R_n}{R_n - 1}\frac{R_b - 1}{R_b}(\alpha - \nu)Al_n^{1-\alpha}\right]^{\frac{\alpha}{1-\alpha}}} < \frac{c_n^2(1+\gamma)^2 R_n^2}{\sigma}\left[1 - \frac{\{1-(1-\psi)R_n^F\}\gamma(R_n - 1)l}{(\alpha - \nu)R_n(1-l)}\right]. \quad (52)$$

In addition, the solution to (37)–(51) must satisfy:

$$\xi^B < 1, \quad (53)$$

$$w_n \geq \sigma, \quad (54)$$

$$\text{all variables are non-negative.} \quad (55)$$

If conditions (52), (53), (54), and (55) are satisfied, the solution to (37)–(51) constitutes a sunspot equilibrium with the probability of a crisis equal to ε.

A.1.1 Deposit guarantee policy

Now suppose that bank deposits are guaranteed by the government. Because the profits of banks and firms are zero when $s = b$, their profit maximization problem is of the same form as in (6) and (12). Remember that the conditions for the laissez-faire equilibrium are given by (37)–(51). With $\tilde{\xi}^B(s) = 1$ for all $s \in \Omega$, conditions (37), (43), and (51) should be replaced by the following equations:

$$R_n^F = 1 + \frac{\psi\varepsilon\frac{c_n}{c_b}}{1-\varepsilon+(1-\psi)\varepsilon\frac{c_n}{c_b}}, \tag{56}$$

$$R_n = \left[\beta\left\{1-\varepsilon+\varepsilon\frac{c_n}{c_b}\right\}\right]^{-1}, \tag{57}$$

$$\xi^B = 1. \tag{58}$$

Conditions (38)–(42), (44)–(50), and (56)–(58) provide a candidate of the sunspot equilibrium.

The consistency condition that guarantees $W = l = 0$ in $s_t = b$ becomes $\frac{1}{\xi^F} > \overline{R}^F$, where $\overline{R}^F = \frac{(1-\alpha)A}{\sigma}\left(\frac{\alpha A}{r_n+q_n-q_b}\right)^{\frac{\alpha}{1-\alpha}}$. Thus the condition corresponding to (52) is given by

$$(1+\gamma)c_n R_n > (1-\alpha)A\left(\frac{\alpha A}{r_n+q_n-q_b}\right)^{\frac{\alpha}{1-\alpha}}. \tag{59}$$

Thus, the solution to (38)–(42), (44)–(50), and (56)–(58) is indeed a sunspot equilibrium with the crisis probability ε if conditions (59), (54), and (55) are satisfied.

A.2 The equilibrium conditions for the fundamental shock economy

Firms' first-order condition with respect to $k(s_{t-1})$, (8), implies

$$\int_{\underline{s}}^{\infty} \beta\frac{c(s_{t-1})}{c(s_t)}\, dF(s_t)R(s_{t-1})q(s_{t-1}) = \int_{\underline{s}}^{\infty} \beta\frac{c(s_{t-1})}{c(s_t)}\{r(s_t)+q(s_t)\}\, dF(s_t). \tag{60}$$

It follows that the term $R(s_{t-1})e(s_{t-1})$ is a constant that is independent of s_{t-1}. The equilibrium condition (19) for the returns on the bank equity and deposits becomes

$$\int_{\underline{s}}^{\infty} \frac{1}{c(s)}\frac{\psi R^F(s)}{1-(1-\psi)R^F(s)}\, dF(s) = \int_{\underline{s}}^{\infty} \frac{1}{c(s)}\, dF(s) + \int_0^{\underline{s}} \frac{1}{c(s)}\xi^B(s)\, dF(s). \tag{61}$$

Since $\overline{s} < \underline{s}$, the net worth constraint of banks (14) binds for all $s_t \in \Omega^n$:

$$W(s_t) = w(s_t)l(s_t) = \frac{R(s_{t-1})e(s_{t-1})}{1-(1-\psi)R^F(s_t)}, \quad \text{for } s_t \in \Omega^n. \tag{62}$$

For $s_t \in \Omega^b$, consumption and labor supply are determined as

$$c(s) = c_b = \frac{\sigma}{1+\gamma}, \tag{63}$$

$$h(s) = h = \frac{1}{1+\gamma}. \tag{64}$$

For $s_t \in \Omega^n$,

$$\frac{R^F(s)\gamma A(s)l(s)^{1-\alpha}}{1-l(s)} = (1-\alpha)A(s)l(s)^{-\alpha}, \tag{65}$$

$$c(s) = A(s)l(s)^{1-\alpha}, \tag{66}$$

$$r(s) = (\alpha-\nu)A(s)l(s)^{1-\alpha}, \tag{67}$$

$$w(s) = \frac{\gamma A(s)l(s)^{1-\alpha}}{1-l(s)}. \tag{68}$$

The first-order condition of households with respect to $d^L(s_t)$ and the first-order condition of firms with respect to $k(s_t)$ imply

$$R(s) = \frac{1}{c(s)\beta\left[\int_0^{\underline{s}} \frac{\xi^B(s')}{c(s')}\, dF(s') + \int_{\underline{s}}^{\infty} \frac{1}{c(s')}\, dF(s')\right]}, \quad \text{for } s \in \Omega^n, \tag{69}$$

$$R_b = \frac{1}{c_b\beta\left[\int_0^{\underline{s}} \frac{\xi^B(s')}{c(s')}\, dF(s') + \int_{\underline{s}}^{\infty} \frac{1}{c(s')}\, dF(s')\right]}, \quad \text{for } s \in \Omega^b, \tag{70}$$

$$q(s) = \frac{1}{R(s)} \frac{\int_{\underline{s}}^{\infty} c(s')^{-1}\{r(s') + q(s')\}\, dF(s')}{\int_{\underline{s}}^{\infty} c(s')^{-1}\, dF(s')}, \quad \text{for } s \in \Omega^n, \tag{71}$$

$$q_b = \frac{1}{R_b} \frac{\int_{\underline{s}}^{\infty} c(s')^{-1}\{r(s') + q(s')\}\, dF(s')}{\int_{\underline{s}}^{\infty} c(s')^{-1}\, dF(s')}, \quad \text{for } s \in \Omega^b. \tag{72}$$

It follows from the definition of $\tilde{\xi}^B(s_t)$ and $\tilde{\xi}^F(s_t)$ that

$$\xi^F(s) = 1, \quad \text{for } s \in \Omega^n, \tag{73}$$

$$\xi^F = \frac{q_b}{R(s_{t-1})q(s_{t-1})} = \frac{q_b \int_{\underline{s}}^{\infty} c(s')^{-1}\, dF(s')}{\int_{\underline{s}}^{\infty} c(s')^{-1}\{r(s') + q(s')\}\, dF(s')}, \tag{74}$$

$$\xi^B(s) = 1, \quad \text{for } s \in \Omega^n, \tag{75}$$

$$\xi^B = \frac{\xi^F[R(s_{t-1})q(s_{t-1}) + R^F W_b]}{R^B W_b + R(s_{t-1})[q(s_{t-1}) - e(s_{t-1})]} = \frac{q_b}{R(s_{t-1})q(s_{t-1}) - R(s_{t-1})e(s_{t-1})}. \tag{76}$$

The threshold value $\underline{s}$ is determined by the break-even condition:

$$\begin{aligned} &A(\underline{s})l(\underline{s})^{1-\alpha} - R^F(\underline{s})w(\underline{s})l(\underline{s}) + q(\underline{s}) \\ &\qquad - \frac{1}{\int_{\underline{s}}^{\infty} c(s)^{-1}\, dF(s)} \int_{\underline{s}}^{\infty} c(s)^{-1}\{r(s) + q(s)\}\, dF(s) = 0. \end{aligned} \tag{77}$$

If s_t is below this threshold, all firms go bankrupt and output is exclusively produced via home production.

The equilibrium values of the endogenous variables are determined as the solution to the system of equations (61)-(77), which is solved in the following fashion. Start with a guess on the values of $\underline{s}$ and $R(s_{t-1})e(s_{t-1})$ (remember that $R(s_{t-1})e(s_{t-1})$ is a constant). Then, equations (62)-(68) are used to determine $\{R^F(s), c(s), c_b, l(s), h, r(s), w(s)\}$ as functions of $\underline{s}$ and $R(s_{t-1})e(s_{t-1})$. Given these, equations (69)-(76) determine $\{R(s), R_b, q(s), q_b, \xi^F, \xi^B\}$, again as functions of $\underline{s}$ and $R(s_{t-1})e(s_{t-1})$. Then equation (61) is used to determine the value of $R(s_{t-1})e(s_{t-1})$. Given $R(s_{t-1})e(s_{t-1})$, $e(s)$ is determined by $e(s) = \frac{R(s_{t-1})e(s_{t-1})}{R(s)}$. Thus all relevant variables are described as functions of $\underline{s}$. Finally, the value of $\underline{s}$ is pinned down by the break-even condition for the firm, (77).

A.2.1 Deposit guarantee policy

The conditions for the laissez-faire equilibrium are given by (61)–(77). With deposits guaranteed, (61), (69), (70), and (76) should be replaced by the following equations:

$$\int_{\underline{s}}^{\infty} \frac{1}{c(s)} \frac{\psi R^F(s)}{1-(1-\psi)R^F(s)} \, dF(s) = \int_0^{\infty} \frac{1}{c(s)} \, dF(s), \tag{78}$$

$$R(s) = \frac{1}{c(s)\beta \left[\int_0^{\infty} \frac{1}{c(s')} \, dF(s')\right]}, \quad \text{for } s \in \Omega^n, \tag{79}$$

$$R_b = \frac{1}{c_b\beta \left[\int_0^{\infty} \frac{1}{c(s')} \, dF(s')\right]}, \quad \text{for } s \in \Omega^b, \tag{80}$$

$$\xi^B(s) = 1 \quad \text{for all } s \in \Omega. \tag{81}$$