Bubbly Dynamics, Land Price, and Welfare

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First Version, December 2010
This Version, December 2011

Abstract

We investigate how appearance of bubble influences the other asset price, especially the land price. We construct a dynamic model in which land is not only a factor of production, but also serves as collateral for loans. The dynamic interaction between bubble price, land price, and output generates powerful bubbly dynamics. The boom-bust of bubble asset price causes boom-crash cycles in the land market simultaneously like contagion by affecting fundamentals of land. We also numerically analyze welfare effects of bubble on transitional dynamics. We show that bubble is welfare-improving.

Key words: Bubbly Dynamics, Fundamentals, Welfare Effects of Bubble

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1 Introduction

Many countries have experienced bubble like dynamics. Boom-bust of asset price bubbles in equity markets tends to be associated with boom-crash of land markets or housing markets, which in turn affects real economic activity very largely. Notable examples include the recent U.S. experiences before and after the financial crisis of 2007-2008 or Japan’s experiences from the late 1980s to the beginning of 1990s.\(^1\)

In this paper, we present a theoretical model in which boom-bust of bubble asset price has contagious effects on another asset price, especially land price, and generates powerful boom-crash effects on macroeconomic performance. The basic structure of our model is as follows. Consider an economy in which there are two types of entrepreneurs—high and low productive entrepreneurs. The type of the entrepreneurs changes over time, which means that an entrepreneur who has high (low) production technology today may have low (high) production one in the future. Both entrepreneurs produce homogeneous goods by using land and intermediate goods. Land is used as collateral for loans as in Kiyotaki and Moore (1997). Besides we assume that there is a bubble asset, which is intrinsically useless in production, i.e., fundamental value of the asset is zero. If the bubble price is zero, the credit constraint for high productive entrepreneurs is tight, so that in equilibrium even low productive entrepreneurs as well as high productive entrepreneurs produce. Resource allocation is inefficient. As a result, output, investment, consumption, TFP, and land price are all low in the steady-state bubbleless economy. On the other hand, in the case of the equilibrium where the bubble price is positive, only high productive entrepreneurs produce using land and more output is produced. This is because high productive entrepreneurs who was low productive entrepreneurs in previous period can sell the bubble asset and increase their net worth, thus being able to buy more land and intermediate goods for their production in next period. Low productive entrepreneurs buy the bubble asset from high productive entrepreneurs instead of producing. Together with this increase in output, the net worth of high and low productive entrepreneurs improves, and their demand for land and their demand for bubble also increase. As a result, both land price

\(^1\)Japan has experienced large movements in equity prices from the late 1980s to the beginning of 1990s. Associated with the boom-bust of equity prices were large fluctuations in land prices, which affected investment, consumption, and output very largely. See Ueda (2011) for Japan’s experiences before and after the bubble’s collapse.
and bubble price rise, reinforcing each other. Because of this, the net worth of H-entrepreneurs improves even further, which increases output, the net worth, land price and bubble price in period after next. These knock-on effects will continue in the future. Moreover, this anticipated increase in land price and bubble price in the future gives positive feedback to land price and bubble price in current period. In equilibrium, all these mechanisms occur simultaneously, which generates bubbly dynamics.

Figure 1 captures these mechanisms. Output, land price, and bubble price interact with each other not only within a period, but also between periods. This dynamic interaction between asset prices and aggregate quantity are similar to Kiyotaki and Moore model (1997). The key innovation of our paper is that the presence of bubble enhances this interaction, thus increasing investment, output, consumption, and land price compared to the bubbleless economy. Once bubble collapses, a reversal mechanism operates. Investment, output, consumption, and land price all fall down eventually. The boom-bust cycles of bubble on a particular financial asset causes boom-crash cycles in the land market simultaneously like contagion.

A feature of equilibrium is that the increase in the land price reflects improvement in fundamentals (cash flow from land), and this improvement is supported by the presence of bubble. This means that bubble changes fundamentals itself.

Although bubble is expansionary in macroeconomic variables, is it welfare-improving or welfare-reducing? Tirole (1985), a seminal paper on rational bubble, shows that bubble is welfare-improving. In his framework, although bubble increases consumption, it decreases investment and output. In the present paper, we investigate welfare effects in a framework in which bubble is expansionary in three variables. In this respect, our paper is related to Kocherlakota (2009). The difference between his analysis and ours is that we consider welfare effects including transitional dynamics, while he focuses on welfare in the steady-state.

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2 This traditional view has been criticized, because it seems inconsistent with empirical evidences during bubbly episodes.
3 Grossman and Yanagawa (1993) develop an endogenous growth model, and show that bubble is welfare-reducing. In their framework, bubble crowds investment out and reduces long run economic growth.
Our paper is related to a number of the recent theoretical development on asset price bubbles such as Farhi and Tirole (2009), Wang and Wen (2009), Hirano and Yanagawa (2010a, 2010b), Martin and Ventura (2010), Sakuragawa (2010) and Aoki and Nikolov (2011). Different from these studies, our model has a fixed asset, land. We investigate how the emergence and collapse of asset price bubble on a particular financial asset affects the other asset price, especially land price.

Kocherlakota’s model (2009) also has land. However, in his model, land is not used as a factor of production and does not produce any output, i.e., fundamental value of the land is zero. He analyzes positive land price, which is bubble. On the other hand, in our model, land is used as a factor of production.

Miller and Stiglitz (2010) are closely related to ours, in the sense that they consider the effects of boom-bust cycles of bubbles in a dynamic model in which land is used as collateral. They describe asset bubble as corrective error of forecast on land price. On the other hand, our paper is based on a rational bubble model.

The paper is organized as follows. In section 2, we first present a basic model, and then derive the existence condition of bubble. We also compare the bubble economy to the bubbleless economy in the steady-state. In section 3, we analyze bubbly dynamics, and discuss welfare implications of bubble.

2 The Model

2.1 Entrepreneur’s Problem

Consider a discrete-time economy with one homogeneous goods and a continuum of entrepreneurs. A typical entrepreneur has the following expected discounted utility,

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log c^i_t \right],$$  

(1)

where $i$ is the index for each entrepreneur, and $c^i_t$ is the consumption of him/her in period $t$. $\beta \in (0, 1)$ is the subjective discount factor, and $E_0 [a]$ is the expected value of $a$ conditional on information at the beginning of period 0.
In each period, each entrepreneur has high production opportunities to produce the homogeneous goods (hereinafter H-projects) with probability \( p \), and low production ones (L-projects) with probability \( 1 - p \). Both high and low productive entrepreneurs (hereafter H-entrepreneurs and L-entrepreneurs) use land and intermediate goods as inputs to produce the homogeneous goods. The land has a fixed supply, which is normalized to be one. The intermediate goods fully depreciates in one period after production. The production technologies are as follows:

\[
y_{i,t+1} = \alpha_i^t \left( \frac{k_i^t}{\sigma} \right)^\sigma \left( \frac{z_i^t}{1 - \sigma} \right)^{1 - \sigma},
\]

where \( k_i^t (\geq 0) \) is land, \( z_i^t (\geq 0) \) is intermediate goods in period \( t \), and \( y_{i,t+1} \) is output in period \( t + 1 \). \( \alpha_i^t \) is productivity in period \( t \). \( \alpha_i^t = \alpha_H \) if the entrepreneur has H-projects, and \( \alpha_i^t = \alpha_L \) if he has L-projects. We assume \( \alpha_H > \alpha_L \). The probability \( p \) is exogenous, and independent across entrepreneurs and over time. At the beginning of each period \( t \), the entrepreneur knows his/her own type in period \( t \), whether he/she has H-projects or L-projects. Assuming that the initial population measure of H-type and L-type is \( p \) and \( 1 - p \), the population measure of each type after period 1 is \( p \) and \( 1 - p \), respectively.

In this economy, there is a bubble asset. We define bubble asset as the asset that produces no real return, i.e., the fundamental value of the asset is zero. Let \( P_t \) be the per unit price of bubble asset in period \( t \) in terms of consumption goods. Then, the entrepreneur’s flow of funds constraint is given by

\[
c_i^t + q_t(k_i^t - k_{i-1}^t) + z_i^t + P_t(x_i^t - x_{i-1}^t) + r_t b_{t-1}^i + q_t \gamma k^t_{i-1} = y_i^t + b_i^t,
\]

where \( x_i^t \) be the level of bubble asset purchased by type \( i \) entrepreneur in period \( t \). The left hand side of (3) is expenditure on consumption, net purchase of land, investment of intermediate goods, net purchase of bubble asset, repayment, and maintenance costs. As in Lorenzoni (2008), we assume that in order to keep the land productive, the entrepreneur has to pay maintenance costs for a fraction \( \gamma \) of his/her land holdings. The right hand side is the

\[4\] A similar setting is used in Woodford (1990), Kiyotaki (1998), Kiyotaki and Moore (2008), and Kocherlakota (2009). In Woodford (1990), the entrepreneurs have investment opportunities in alternating periods.
available funds in period $t$, which is the return from investment in the previous period and new borrowing. We define the net worth of the entrepreneur in period $t$ as $e_{it} \equiv y_{it} - r_{t-1}b_{t-1} + q_{t}k_{t-1}^{i}(1 - \gamma) + P_{t}x_{t-1}^{i}$.

We assume that because of frictions in a financial market, the entrepreneurs are credit-constrained. Following Kiyotaki and Moore (1997), creditors limit credit so that debt repayment cannot exceed the value of collateral, i.e., the value of the land minus maintenance costs. That is, the borrowing constraint becomes:

$$r_{t}b_{t}^{i} \leq q_{t+1}k_{t}^{i} - q_{t+1}\gamma k_{t}^{i},$$

where $q_{t+1}$ is the price of land in period $t + 1$. $r_{t}$ and $b_{t}^{i}$ are the gross interest rate and the amount of borrowing in period $t$, respectively.

We also impose short sale constraint on bubble asset:5

$$x_{t}^{i} \geq 0.$$  

2.2 Equilibrium

Let us denote aggregate consumption of H-and L-entrepreneurs in period $t$ as $\sum_{i \in H_{t}} c_{i}^{t} \equiv C_{t}^{H}$, $\sum_{i \in L_{t}} c_{i}^{t} \equiv C_{t}^{L}$, where $H_{t}$ and $L_{t}$ mean a family of H- and L-entrepreneurs in period $t$. Similarly, let $\sum_{i \in H_{t}} z_{i}^{t} \equiv Z_{t}^{H}$, $\sum_{i \in L_{t}} z_{i}^{t} \equiv Z_{t}^{L}$, $\sum_{i \in H_{t}} b_{i}^{t} \equiv B_{t}^{H}$, $\sum_{i \in L_{t}} b_{i}^{t} \equiv B_{t}^{L}$, $\sum_{i \in H_{t}} k_{i}^{t} \equiv K_{t}^{H}$, $\sum_{i \in L_{t}} k_{i}^{t} \equiv K_{t}^{L}$, $\sum_{i \in H_{t} \cup L_{t}} x_{i}^{t} \equiv X_{t}$, $\sum_{i \in H_{t} \cup L_{t}} y_{i}^{t} \equiv Y_{t}$ be aggregate investment, aggregate borrowing, aggregate land holdings, and aggregate demand for bubble asset of each type. Assuming that aggregate supply of bubble asset is fixed over time $X$, then the market clearing condition for goods, credit, land, and bubble asset are

$$C_{t}^{H} + C_{t}^{L} + Z_{t}^{H} + Z_{t}^{L} = Y_{t} - q_{t}\gamma,$$

$$B_{t}^{H} + B_{t}^{L} = 0,$$

$$K_{t}^{H} + K_{t}^{L} = 1,$$

5Kocherlakota (1992) shows that the short sale constraint plays an important role for the emergence asset price bubbles in an infinitely lived agent model.
The competitive equilibrium is defined as a set of prices $\{r_t, q_t, P_t\}_{t=0}^{\infty}$ and quantities $\{c_t^i, b_t^i, z_t^i, x_t^i, k_t^i, C_t^H, C_t^L, B_t^H, B_t^L, Z_t^H, Z_t^L, K_t^H, K_t^L, Y_t\}_{t=0}^{\infty}$, such that (i) the market clearing conditions, (6)-(9) are satisfied, and (ii) each entrepreneur chooses consumption, borrowing, land holdings, investment of intermediate goods, and bubble asset to maximize his/her expected discounted utility (1) under the constraints (2), (3), (4), and (5). Because there is no aggregate uncertainty, the entrepreneurs have perfect foresight of future prices and aggregate quantities in equilibrium. We also rule out the exploding bubbles in land price:

$$\lim_{t \to \infty} q_t^{\gamma} < 0.$$

As is well known, there are multiple equilibria in the rational bubble model. One is the equilibria with $P_t > 0$, and the other is $P_t = 0$. In the rest of the analysis, we focus on $P_t > 0$ and analyze what happens in the bubble economy.

### 2.3 Entrepreneur’s Behaviour

We are now in a position to characterize the equilibrium behaviour of entrepreneurs in the bubble economy. We consider the case

$$\frac{\alpha^L}{u_t} < r_t < \frac{\alpha^H}{u_t},$$

where $u_t = q_t - q_{t+1}(1-\gamma)/r_t$ is the opportunity cost, or user cost, of holding land from $t$ to $t+1$. $\alpha^L/u_t$ and $\alpha^H/u_t$ are the rate of return of L-projects and H-projects, respectively.

In order that bubble asset be held in equilibrium, the rate of return of bubble has to be equal to interest rate:

$$r_t = \frac{P_{t+1}}{P_t}. \quad (12)$$

When (11) and (12) hold, both the borrowing constraint and the short sale constraint simultaneously become binding for the entrepreneurs who have H-projects in period $t$, but not binding for the entrepreneurs who
have L-projects. Since the utility function is log-linear, each entrepreneur consumes a fraction \(1 - \beta\) of the net worth in each period, that is, 
\[c_t^i = (1 - \beta) \left[ y_t^i - r_{t-1}b_{t-1}^i + q_t^i k_{t-1}^i (1 - \gamma) + P_t x_{t-1}^i \right].\]
Then, by using (3), (4), and (5), we can derive demand functions of the type \(i\) agent of H-entrepreneurs for intermediate goods and land holdings in period \(t\):

\[z_t^i = (1 - \sigma) \beta \left[ y_t^i - r_{t-1}b_{t-1}^i + q_t^i k_{t-1}^i (1 - \gamma) + P_t x_{t-1}^i \right],\]

\[k_t^i = \frac{\sigma \beta \left[ y_t^i - r_{t-1}b_{t-1}^i + q_t^i k_{t-1}^i (1 - \gamma) + P_t x_{t-1}^i \right]}{q_t^i - q_{t+1}^i (1 - \gamma) / r_t}.\]

Equation (13) says that H-entrepreneur spends a fraction \(1 - \sigma\) of his/her saving on intermediate goods. Equation (14) says that H-entrepreneur spends a fraction \(\sigma\) of his/her saving to finance the difference between the land value \(q_t^i\) and the collateral value \(q_t^i (1 - \gamma) / r_t\). The difference \(q_t^i - q_{t+1}^i (1 - \gamma) / r_t\) is thought of as the downpayment to purchase one unit of land, which happens to be the user cost \(u_t\). From (13) and (14), we understand that for the entrepreneurs who purchased bubble asset in the previous period, they are able to sell the asset at the time they encounter H-projects. In our analysis, the entrepreneurs buy bubble whey they have L-projects, and sell those bubble when they have a opportunity to invest in H-projects. As a result, their net worth increases (compared to the bubbleless case), which boosts their demand for intermediate goods and land.

For L-entrepreneurs in period \(t\), they prefer buying bubble and lending to H-entrepreneurs instead of investing in their own L-projects when (11) and (12) hold, because the rate of return of bubble or that of lending is greater than the rate of return of L-projects.

### 2.4 Aggregation

Since (13) and (14) are linear functions of the net worth, we can aggregate across H-entrepreneurs to derive the aggregate demand functions:

\[Z_t^H = (1 - \sigma) \beta p [Y_t + q_t (1 - \gamma) + P_t X],\]

\[6\text{See, for example, chapter 1.7 of Sargent (1988).}\]
where $Y_t + q_t(1 - \gamma) + P_t X$ is aggregate wealth of the entrepreneurs. Since every entrepreneur has the same chance to meet H-projects in each period, a fraction $p$ of aggregate wealth is the wealth of H-entrepreneurs. Equation (15) and (16) say that the aggregate demand functions for intermediate goods and land depend upon cash flow from investments in the previous period $Y_t$ as in Bernanke and Gertler (1989) and land price $q_t$ as in Kiyotaki and Moore (1997). What is new in our framework is that the demand functions also depend upon bubble price $P_t$, i.e., the presence of bubble crowds in demands of land holdings and intermediate goods, not crowds out them as in traditional literature (Tirole (1985)).

Since only H-entrepreneurs use land, from the land market clearing condition,

$$K_t^H = \frac{\sigma \beta p}{q_t - \frac{q_{t+1}(1 - \gamma)}{r_t}} \left[ Y_t + q_t(1 - \gamma) + P_t X \right],$$

(16)

Notice that $q_t$ is an increasing function of $P_t$, $Y_t$, and $q_{t+1}$. When $P_t$ increases, the net worth of H-entrepreneurs improves. As a result, they can buy more land with maximum leverage, which raises the current land price $q_t$. Moreover, when $q_{t+1}$ is expected to increase, the borrowing constraint is relaxed, which increases the demand for land as in Kiyotaki and Moore model (1997). In our model, $q_{t+1}$ is affected by bubble price in period $t + 1$, $P_{t+1}$.

From the aggregate flow of funds constraint of L-entrepreneurs and the bubble market clearing condition (9), we can derive equilibrium bubble price in period $t$:

$$P_t X = \frac{\beta(1 - p) \left[ Y_t + q_t(1 - \gamma) \right] - B_t^H}{1 - \beta + p \beta},$$

(18)

We see that given $B_t^H$, $P_t$ is an increasing function of $Y_t$ and $q_t$. Intuitively, when cash flow or land price increases, the net worth of L-entrepreneurs improves and they can buy more bubble, which raises bubble price.

Notice from (17) and (18), bubble price $P_t$ and land price $q_t$ reinforce each other within a period. Moreover, as we will see, since both asset prices are forward-looking variables, they interact with each other between periods.

Since only H-entrepreneurs produce by using land and intermediate goods, output evolves over time as
\[ Y_{t+1} = \frac{\alpha^H}{\sigma} u_t^{1-\sigma}. \] (19)

From the definition of the user cost of land and equation (12), the land price should satisfy the dynamic equation:

\[ q_t = u_t + \frac{P_t}{P_{t+1}} q_{t+1}(1 - \gamma). \] (20)

From this, the current land price equals the discounted value of future user costs, which in turn depends upon the discounted value of future output from (19). This means that when output is expected to increase in the future, the current land price rises. And, future output is affected by the presence of bubble.

Since both H-and L-entrepreneurs consume a fraction \( 1 - \beta \) of the net worth, goods market clearing condition (6) can be written as

\[ (1 - \beta) A_t + \frac{1 - \sigma}{\sigma} u_t = Y_t - q_t \gamma, \] (21)

where \( A_t \equiv Y_t + q_t(1 - \gamma) + P_t X \) is aggregate wealth. The first term of the left hand side of (21) is aggregate consumption and the second term is investment of intermediate goods.

Along the perfect foresight equilibrium path, aggregate wealth evolves over time as

\[ A_{t+1} = \frac{\alpha^H}{u_t} p \beta A_t + \frac{P_{t+1}}{P_t} (1 - p) \beta A_t. \] (22)

Equation (22) says that given the aggregate saving of the economy \( \beta A_t \), the aggregate net worth of H-entrepreneurs earns a rate of return \( \alpha^H/u_t^\sigma \), while the net worth of L-entrepreneurs earns \( P_{t+1}/P_t \).

**Proposition 1** Output, investment, consumption, and aggregate TFP are all higher in the bubbly steady-state than in the bubbleless steady-state. 

**Proof.** Proof is in Appendix. ■

In the steady-state of the bubbleless economy, \(^7\) production is inefficient, in the sense that L-entrepreneurs as well as H-entrepreneurs produce by using

\(^7\)Mathematical description in the bubbleless economy is in proof of Proposition 3 in Appendix.
land and intermediate goods, which means that resource allocation is inefficient. On the other hand, in the bubbly steady-state, only H-entrepreneurs produce. This means that bubble improves efficiency in production by eliminating inefficient production, L-projects. In other words, the presence of bubble helps transfer resources from L-entrepreneurs to H-entrepreneurs. Since the resource allocation is more efficient, all macroeconomic variables are higher than those in the bubbleless steady-state.

With regard to the effects of bubble on the land price, since only H-entrepreneurs produce in the bubbly steady-state, cash flow from land increases from the present to the future, which raises the steady-state land price. On the other hand, interest rate is high in the bubbly steady-state, which decreases the land price. In this sense, there are two competing effects of bubble on the land price. Which one of these effects dominates depends upon the value of $\gamma$. In the following numerical example, we focus on the case where bubble increases the land price in the steady-state.

An interesting feature in our framework is that bubble changes fundamentals itself, i.e., cash flow from land and interest rate. When the land price is higher in the bubble economy, it reflects the improvement in cash flows, and this improvement itself is supported by the presence of asset bubble. This implies that boom-bust cycles of bubble on a particular financial asset is associated with dramatic changes in fundamentals in the other market (land market).

### 2.5 Existence of Asset Bubble

In order that bubble can exist in the steady-state, the following two conditions must be satisfied:

$$\frac{\alpha^L}{u^\sigma} \leq 1,$$

(23)

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8In Kiyotaki and Moore (1997), interest rate equals gatherers’ time preference rate and becomes constant over time, even if productivity shocks occur, while in our model, interest rate changes endogenously by whether bubbles occur or not.

9There is a threshold value of $\gamma \equiv \hat{\gamma} < 1$, above which bubbles increase the steady-state land value and below which they decrease it. $\hat{\gamma}$ satisfies the following equation:

$$q = \frac{u}{\gamma} > q' = \frac{r'u'}{r' - (1 - \gamma)},$$

where $u'$ denotes the user cost of holding land in the bubbleless economy.
(23) says that the rate of return of bubble must be greater than the rate of return of L-projects. Otherwise, no entrepreneur buys bubble. (24) says that bubble price must be positive. This second condition is satisfied as long as L-entrepreneurs produce in the bubbleless economy, \( K'_L > 0 \). From these conditions, we get the following Proposition.

**Proposition 2** Bubble can exist if and only if the following condition is satisfied,

\[
\alpha^H \geq \frac{\alpha^L p \beta}{1 - \beta + p \beta},
\]

**Proof.** Proof is given in Appendix. ■

In order that bubble can exist in the steady-state equilibrium, productivity \( \alpha^H \) must be sufficiently high. Intuitively, if productivity is high enough, the marginal productivity of production becomes lower, which means that the user cost becomes sufficiently large, so that the rate of return of bubble becomes greater than the rate of return of L-projects.

We should add a few more remarks on maintenance costs. As in traditional rational bubble literature such as Tirole (1985) and Farhi and Tirole (2011), in the steady-state bubble economy with no growth, interest rate equals one, in which case (10) cannot be satisfied in our model, because our model has a fixed asset, land. In order that (10) is satisfied, we need to introduce costs associated with land holdings.

### 3 Macroeconomic Effects of Asset Bubble

Given an initial condition, \( Y_0 \), the perfect foresight equilibrium path is described by sequences \( \{q_t, A_t, r_t, u_t, P_t, Y_t\}_{t=0}^{\infty} \), satisfying (10), (19), (20), (21), and (22). With regard to this dynamical system, we can get the following Proposition.

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\[^{10}\text{The condition that L-entrepreneurs produce in the bubbleless economy is the following:}

\[1 - p(1 - \sigma) - \sigma p \frac{\alpha^L}{\alpha^L - (1 - \gamma) [\alpha^H p + \alpha^L (1 - p)] \beta} > 0.\]

If this condition is satisfied, then, \( P_t > 0 \) holds true.
Proposition 3 There is a saddle point path on which the economy converges to a steady-state bubble economy.

Proof. Proof is given in Appendix.

Once the economy gets on the saddle point path, the economy's output dynamics can be described as the following simple difference equation:

$$Y_{t+1} = \frac{\alpha^H}{\sigma} \left( \frac{\sigma p \beta}{1 - \beta + p \beta} \right)^{1-\sigma} (Y_t)^{1-\sigma},$$

and also

$$q_t = \frac{\sigma p \beta Y_t}{\gamma} \left( \frac{1 - \beta + p \beta}{1 - \beta + p \beta} \right),$$

$$P_t X = \left[ 1 - (1 - \sigma)p \right] \frac{\gamma - \sigma p \beta Y_t}{1 - \beta + p \beta},$$

$$u_t = \frac{\sigma p \beta Y_t}{1 - \beta + p \beta}.$$

Since we get the equations of the equilibrium path, we can analyses the output dynamics analytically. In Figure 2, we show the equilibrium dynamics of output in bubble economy and in bubbleless economy. Suppose that the economy is initially in bubbleless steady-state, $Y_{\text{bubbleless}}^*$, and suddenly people believe that the bubble emerges and will exist with positive price forever. The output gets larger and converges to steady state in bubble economy, $Y_{\text{bubble}}^*$.

[Insert Figure 2]

3.1 Numerical Example

We show a numerical experiment on the effects of boom-bust cycles of bubble on macroeconomic variables.\footnote{It is not necessarily needed to show the numerical experiment to show the dynamics of output. However, in order to show the welfare analysis later, it is needed to simulate a numerical example in the transition path.} For $t \leq 0$, the economy is in the steady-state in the bubbleless economy, in which all entrepreneurs believe that the price of bubble asset shall be zero in the future: $P_t = 0$ for all $t$. We suppose that at the beginning of period $t = 1$, the price of bubble asset unexpectedly becomes
positive $P_t > 0$, and all entrepreneurs expect that the bubble will last forever. At the beginning of period $t = 51$, the bubble collapses unexpectedly $P_t = 0$, and once bubble bursts, all expect that the bubble will not emerge at all in the future. Note that there is no aggregate productivity shock for all $t$. The model is solved by the shooting method. The parameter values are set as follows: $\alpha_H = 1; \alpha_L = .8; \beta = .99; \gamma = .2; p = .05; \sigma = .3; X = 1$. Most of these values seem standard.

Figure 3 plots dynamics of the macroeconomic variables when bubble occurs. Consumption rises immediately after the emergence of bubble in period $t = 1$ and continues to increase over time because of wealth effect. Recall that consumption is a fraction $1 - \beta$ of the net worth and the net worth is improved over time. Output remains unchanged in period $t = 1$, because it is predetermined. But in period $t = 2$, it expands because of reallocation of land, i.e., land is used only by H-entrepreneurs in the bubbly economy ($t \geq 1$). After $t = 3$, since bubble continues to improve the net worth of H-entrepreneurs, which comes from high rate of return of bubble, and enhances the crowd-in effect, output, expenditure of intermediate goods, and land price increase over time. We emphasize that the increase in the land price reflects an improvement in fundamentals (or cash flow from land). With regard to land price and expenditure for intermediate goods in period $t = 1$, both of them decline immediately after the emergence of bubble. This is because, when bubble arises, the entrepreneurs buy bubble, which crowds savings away from the purchase of land and intermediate goods.\footnote{Behind this, interest rate rises substantially, reflecting a tightness of the credit market.} This means that in the beginning of bubble’s appearance, a traditional crowd-out effect of bubble dominates the crowd-in effect in the land market and intermediate goods market.

When bubble bursts in period $t = 51$, a reverse mechanism operates. The net worth of entrepreneurs decreases over time and land is used inefficiently, which means that L-entrepreneurs as well as H-entrepreneurs produce. As a result, output, consumption, expenditure of intermediate goods, and land price fall down and converge to the lower, steady-state level.\footnote{Note that the land price and expenditure for intermediate goods increase immediately after the bubble bursts, because all the savings of the entrepreneurs flow to the purchase of land and intermediate goods, by which interest rate falls substantially.} Our simulation result suggests that the boom-bust cycles of bubble on a particular financial asset causes boom-crash cycles in the land price simultaneously like contagion.
Here we add a few remarks on the response of land price and expenditure of intermediate goods. In this numerical example, we investigate pure effects of bubble; the birth and bust of asset bubble is caused by the sudden change in entrepreneurs’ expectations. If the emergence and collapse of bubble is triggered by a change in productivity $\alpha^H$, then land price and expenditure of intermediate goods will increase immediately after the emergence of bubble, and then will decline immediately after the bubble crash.

3.2 Welfare Effects of Bubble

The bubble’s appearance has positive effects on macroeconomic variables. In this subsection, we discuss the welfare effects of asset price bubble. We compare ex-ante welfare of the entrepreneurs under two situations. As in the previous numerical example, until period $t \leq 0$, the economy is in the steady-state of the bubbleless economy. One situation is that at the beginning of period $t = 1$, bubble arises and after period $t \geq 1$, the economy runs on the bubbly economy’s path. The other situation is that bubble never arises for all $t \geq 0$ and the economy continues to stay in the steady-state of the bubbleless economy.

When we compute ex-ante welfare in period $t = 1$,

$$V^i_1 = E_1 \left[ \sum_{t=1}^{\infty} \beta^{t-1} \log c^i_t \right] = E_1 \left[ \sum_{t=1}^{\infty} \beta^{t-1} \log(1 - \beta) e^i_t \right]$$

where $V^i_1$ is ex-ante welfare of the type $i$ entrepreneur in period $t = 1$. Since $e^i_{t+1} = R^i_t \beta e^i_t$, the above equation can be rewritten as

$$V^i_1 = \sum_{t=1}^{\infty} \beta^{t-1} \log(1 - \beta) \beta^{t-1} + \frac{1}{1 - \beta} \log e^i_1 + \frac{\beta}{1 - \beta} \log R^i_1$$

$$+ E_1 \left[ \sum_{t=2}^{\infty} \frac{\beta^t}{1 - \beta} \log R^i_t \right], \quad (30)$$

where in the bubble economy,

$$R^i_t = \begin{cases} \frac{\alpha^H}{\beta^i_t} & \text{if } i = H, \\ \frac{P^i_t}{P^i_1} & \text{if } i = L, \end{cases}$$
where in the bubbleless economy, 
\[ R_t = \begin{cases} \frac{\alpha_H}{u_t'} & \text{if } i = H, \\ \frac{\alpha_L}{u_t'} & \text{if } i = L. \end{cases} \]

\( u' \) denotes the user cost of holding land in the bubbleless economy.

We make two assumptions to compute ex-ante welfare in period \( t = 1 \). The first one is that at the beginning of period \( t = 1 \), each entrepreneur is endowed with \( X \) unit of bubble asset. The second assumption is that we define the welfare as sum of the third term and the fourth term in (31). When bubble occurs at the beginning of period \( t = 1 \), the bubble price jumps up, but land price jumps down as shown in the numerical example. Hence, the effect of bubble’s emergence on the net worth in period \( t = 1 \) (the second term in (31)) is ambiguous theoretically. However, in our numerical example, every individual’s net worth in period \( t = 1 \) improves. Given these assumptions, when bubble arises, each individual’s net worth certainly increases and the first two terms in (31) also increase. It is no wonder that the effect of bubble’s emergence raises the welfare naturally, if we include the second term in (31) as the welfare. Since the first term is the same for all entrepreneur, it is not needed to compare the level of welfare. Therefore, we exclude those term from the definition of welfare.

When computing them in the bubble economy, we take into account transitional dynamics from the steady-state of the bubbleless economy to the steady-state of the bubble economy. The parameter values are the same as before.

[Insert Figure 4]

Figure 4 plots the values of expected welfare for H and L type individuals with respect to \( p \). The first raw shows the expected welfare for the bubble economies, while the second raw shows the expected welfare for the bubbleless economies. (need to show the intuition for this result.)

[Insert Figure 5]

Figure 5 shows the difference of expected welfare for H and L type between bubble and bubbleless economy, which describe the appearance of bubble is welfare-improving for both type individuals. Intuitively, this comes from the
difference in rate of return. Without bubble, L-entrepreneurs end up with accumulating their wealth through a low return savings vehicle, whose rate of return is $\alpha^L/u_t^\nu$. On the other hand, bubble provides a high rate of return vehicle for them, $P_{t+1}/P_t$. This change in the rate of return contributes to improve welfare of entrepreneurs.

4 Conclusion

We have constructed a model of dynamic economy in which land is not only a factor of production, but also serves as collateral for loans. We have investigated how appearance of bubble influences the other asset price, especially land price. The dynamic interaction between bubble price, land price, and output generates powerful bubbly dynamics. The boom-bust cycles of bubble asset price on a particular financial asset causes boom-crash cycles in the land market simultaneously like contagion by affecting fundamentals of land. We have also numerically analyzed welfare effects of bubble on transitional dynamics. We find that bubble is welfare-improving.

Our framework can be extended in several directions. Let us just discuss one of them here. In the above model, we have analyzed interaction between asset price bubble in a particular financial asset and the other asset price in a closed economy. A promising direction is to include a two-countries model or a multi-countries model. By so doing, we will be able to analyze how boom-bust cycles of asset price bubble in one country has contagious effects on asset prices in different countries. This analysis will be important to understand the recent global financial crisis and debt crisis across countries.

References


Figure 1: Mechanism of Bubbly Dynamics
Figure 2: Bubbly Dynamics of Output
Figure 3: Effects of Boom-Bust of Bubble Asset Price
Figure 4: Expected Welfare for Each Individual
Figure 5: Difference of Expected Welfare between Bubble and Bubbleless Economy
Appendices

A  Proof of Proposition 1

In order to prove Proposition 1, we first characterize an equilibrium of the bubbleless economy. When there is no bubble, interest rate, user cost, output, and wealth evolves over time as

\[ r_t' = \frac{\alpha^L}{u_t^\sigma}, \quad (A1) \]

\[ q_t = u_t + \frac{u_t^\sigma}{\alpha^L} q_{t+1}(1 - \gamma) \quad (A2) \]

\[ Y_{t+1}' = \frac{\alpha^H}{\sigma}(u_t')^{1-\sigma}K_t'' + \frac{\alpha^L}{\sigma}(u_t')^{1-\sigma}K''_L, \quad (A3) \]

\[ A_{t+1}' = \frac{\alpha^H}{u_t^\sigma} \beta p A_t' + \frac{\alpha^L}{u_t^\sigma} \beta(1 - p) A_t'. \quad (A4) \]

In the bubbleless economy, interest rate equals the rate of return of L-projects, so that even L-entrepreneurs end up with producing in equilibrium, which means that \( K^{tL} \geq 0 \) and \( Z^{tL} \geq 0 \).

Given an initial condition, \( Y_0 \), and \( P_t = 0 \) for all \( t \geq 0 \), the perfect foresight equilibrium path of the bubbleless economy is described by sequences \( \{q_t, A_t, r_t, u_t, Y_t\}_{t=0}^{\infty} \), satisfying (10), (21), and (A1)-(A4).

In the steady-state, the user cost is

\[ u^\sigma = \alpha^H \beta p + \alpha^L \beta(1 - p). \quad (A5) \]

The condition (25) is equivalent to

\[ u^\sigma \leq u''^\sigma. \quad (A6) \]

This means that as long as bubble can exist, the user cost in the steady-state bubble economy is greater than that in the steady-state bubbleless economy.

Since \( u^\sigma \geq u''^\sigma \) and \( K^H \geq K''^H \), from (19) and (A3),

\[ Y \geq Y'. \quad (A7) \]
Moreover, $u^\sigma \geq u'^\sigma$ and $K^H \geq K'^H$ mean

$$\frac{uK^H}{\sigma} \geq \frac{u'K'^H}{\sigma}. \quad (A8)$$

Hence,

$$Z^H \geq Z'^H, \quad (A9)$$

$$A \geq A', \quad (A10)$$

because in equilibrium, $uK^H/\sigma = Z^H/1 - \sigma = \beta p A$ and $u'K'^H/\sigma = Z'^H/1 - \sigma = \beta p A'$ hold.

We also know that aggregate consumption is a fraction $1 - \beta$ of the aggregate wealth. Hence,

$$C \geq C'. \quad (A11)$$

With regard to aggregate TFP, it can be defined as

$$\text{TFP} \equiv \frac{Y}{\left(\frac{K^H}{\sigma}\right)^{\sigma - (1 - \sigma)}}, \quad \text{TFP'} \equiv \frac{Y'}{\left(\frac{K'^H}{\sigma}\right)^{\sigma - (1 - \sigma)}}, \quad (A12)$$

where $Z = Z^H + Z^L$ and $Z' = Z'^H + Z'^L$. Hence,

$$\text{TFP} = \alpha^H \geq \text{TFP'} \equiv \frac{\alpha^H Z^H - \alpha^L Z^L}{Z'} \quad (A13)$$

### B Proof of Proposition 2

In the steady-state bubble economy, from (22), $u^\sigma$ is

$$u^\sigma = \frac{\alpha^H p \beta}{1 - \beta + p \beta}. \quad (B1)$$

By substituting (B1) into (23), we can derive (25).

### C Proof of Proposition 3

By substituting (17), (21), and (22) into (20), we obtain

$$\phi_t = \sigma p + \frac{\delta - \phi_t}{\delta - \phi_{t+1}} \phi_{t+1} (1 - \gamma), \quad (C1)$$
where $\phi_t \equiv q_t/\beta A_t$.

Given a state variable $Y_t$, there is a unique price path $\{q_t, P_t\}_{t=0}^\infty$ where $\phi_t$ becomes constant over time and satisfies

$$\phi = \sigma p/\gamma.$$  \hfill (C2)

Once $\phi_t$ becomes constant, then we can derive (27)-(29) from (21) and (C2). And by using (17), (19) can be rewritten as

$$Y_{t+1} = \frac{\alpha^H}{\sigma} (\sigma \beta p A_t)^{1-\sigma}. \hfill (C3)$$

(C3) can be rearranged as (26) by using (27), (28), and the definition of $A_t$. Hence, the economy converges to the steady-state according to (26), and (27)-(29). Initial values of $\{q_0, P_0\}$ are determined so that the economy gets on the saddle path.