LEVERAGE AND PRODUCTIVITY

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\textsuperscript{1}These views are those of the author and do not necessarily reflect the views of the Federal Reserve System.
1. Can more productive firms borrow more?

2. What is the aggregate productivity loss due to financial frictions?
FINDINGS

1. For young and unlisted firms in Japan:
   - leverage rises almost one-for-one with productivity
   - output-to-capital ratio rises strongly with productivity

2. Implications within a standard macro framework:
   - more productive firms can borrow more
   - the constant leverage model overstates aggregate productivity loss from financial frictions by 30%
Frictionless

Leverage vs Return to capital

Productivity vs Productivity
CONSTANT LEVERAGE
Frictionless vs Constant leverage

Productivity
Leverage

Constant leverage Frictionless

Misallocation
Return to capital

Productivity
Leverage
**Polar Models versus Data**

<table>
<thead>
<tr>
<th>frictionless</th>
<th>my finding</th>
<th>constant leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>10.1%</td>
<td>13.7%</td>
</tr>
</tbody>
</table>

**Table**: aggregate productivity loss
Related Literature


Contribution

New facts: leverage and output-to-capital ratios rise with productivity for young and unlisted firms in Japan.

Use the new facts to discipline macro models of financial frictions.
Roadmap

- Model
- Quantification
  - Empirical patterns
  - Indirect inference
- Aggregate implications
ROADMAP

- Model
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ENVIRONMENT

A continuum of entrepreneurs

- infinitely-lived, CRRA utility
- can operate one business with Cobb-Douglas production technology
- idiosyncratic productivity
- can save and borrow

$L$ workers, hand-to-mouth, each supply one-unit of labor

Aggregate output equals the sum of entrepreneur’s output. No aggregate shocks.
Entrepreneur’s problem

\[
V(a, z) = \max_{a', c} u(c) + \beta \mathbb{E} \left[ V(a', z') | z \right]
\]

subject to

\[
c + a' \leq a(1 + r) + \pi(a, z)
\]

\[
\ln z' = \rho \ln z + \epsilon, \quad \epsilon \stackrel{iid}{\sim} N(\mu_e, \sigma^2_e)
\]

where

\[
\pi(a, z) := \max_{k, l} z(k^{\alpha}l^{1-\alpha})^\eta - Rk - wl, \quad R := r + \delta
\]

\[
k \leq \bar{k}(a, z)
\]
Policy functions

Unconstrained firms:

\[ k(a, z) \propto z^{\frac{1}{1-\eta}} \quad \eta \alpha \frac{y(a, z)}{k(a, z)} = R \]

Constrained firms:

\[ k(a, z) = \bar{k}(a, z) \quad \eta \alpha \frac{y(a, z)}{k(a, z)} = R + \mu(a, z) \]

\[ R + \mu(a, z) \propto \frac{z^{\frac{1}{1-(1-\alpha)\eta}}}{\bar{k}(a, z)^{\frac{1-\eta}{1-(1-\alpha)\eta}}} \quad \text{if constrained} \]
Financial Constraint

\[ \log k(a, z) \]
(borrowing capacity)

Unconstrained
\[ MPK = R \]

Constrained
\[ MPK > R \]

Optimal capital

\[ \log z \] (productivity)
**Collateral constraint**

If default, entrepreneurs keep $1 - \phi_y$ fraction of revenue, $1 - \phi_k$ fraction of depreciated capital, lose all inside equity.

Can use financial market after one period without further penalties.

$\bar{k}(a, z)$ is the maximum capital satisfying

$$\phi_y \max_l \{zf(k, l) - wl\} + \phi_k (1 - \delta)k + (1 + r)a \geq (R + 1 - \delta)k$$

$\phi_y = 1, \phi_k = 1$: unconstrained

$\phi_y = 0$: constant leverage model $k \leq \lambda a$.
A stationary competitive equilibrium consists of labor demand $l(a, z)$, capital demand $k(a, z)$, savings policy, interest rate and wage, wealth and productivity distribution $G(a, z)$ such that

1. given prices, $l(a, z)$, $k(a, z)$ and savings policy solve the entrepreneur’s problem

2. capital market and labor market clear

3. $G(a, z)$ is consistent with the savings policy and the law of motion of $z$.
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Firm-level data

- TSR-Orbis firm level data from Japan, 2004-2013
- TSR: Japan’s largest credit rating agency
- unlisted limited liability companies and corporations
- age = years since incorporation
- unbalanced panel
## Coverage

<table>
<thead>
<tr>
<th>incorp year</th>
<th>TSR-Orbis</th>
<th>Census¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>8,995</td>
<td>35,114</td>
</tr>
<tr>
<td>2006</td>
<td>9,826</td>
<td>28,946</td>
</tr>
<tr>
<td>2011,2012</td>
<td>9,405</td>
<td>21,312</td>
</tr>
</tbody>
</table>

¹ single unit or main companies establishments

**Table:** Company counts. TSR-Orbis, Census
Entrant shihonkin distribution compared to the Census

![Shihonkin distribution of firms incorporated in 2006](image)
Entrant workforce distribution compared to the Census

[Graph showing workforce distribution of firms incorporated in 2006, comparing Census and TSR-Orbis data]
**Definition of variables**

- $k = \text{book value of capital stock (total asset)}$
- $y = \text{operating revenue} \times (1 - \text{factor share of materials})$
- $l = \text{number of employees}$
- $\ln z = \ln y - \eta \alpha \ln k - \eta (1 - \alpha) \ln l$

  $\eta = 0.85, \ \alpha = \text{factor share of capital in value added}$

- factor shares from JIP Database 2013, average over 2000-2010, 108 sectors
- $a = \text{shihonkin} \ or \ shareholders \ fund$

..
Firm leverage rises with firm productivity.
Firm output-capital ratio rises with firm productivity

![Graph showing the relationship between residualized log output capital ratio and residualized log productivity. The graph demonstrates an increasing trend as productivity increases.](image-url)
# Regressions on Log Productivity

<table>
<thead>
<tr>
<th>Dep. Var</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>leverage</td>
<td>linear</td>
<td>quad.</td>
<td>diff $a$</td>
<td>diff $k$</td>
<td>firm FE</td>
<td>2SLS</td>
</tr>
<tr>
<td></td>
<td>1.125</td>
<td>1.120</td>
<td>0.973</td>
<td>0.346</td>
<td>0.489</td>
<td>0.207</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.050)</td>
<td>(0.040)</td>
<td>(0.051)</td>
<td>(0.034)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>output-capital</td>
<td>0.690</td>
<td>0.598</td>
<td>0.751</td>
<td>1.316</td>
<td>1.126</td>
<td>3.205</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.035)</td>
<td>(0.032)</td>
<td>(0.039)</td>
<td>(0.034)</td>
<td>(0.684)</td>
</tr>
<tr>
<td>$N$</td>
<td>5872</td>
<td>5872</td>
<td>5872</td>
<td>5870</td>
<td>21962</td>
<td>5872</td>
</tr>
</tbody>
</table>

NAICS 6-digit industry FE. Control for log inside fund. 2SLS use employment to instrument for productivity. 2006 cohort. Age 5 except for (5). Similar results for other year-cohort.
Roadmap

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INDIRECT INFERENCE FOR $\phi_y$ AND $\phi_k$

Target: regression coefficients in the auxiliary model

\[
\begin{align*}
\ln \frac{y}{k} &= \beta_0 + \beta_1 \ln z + \beta_2 (\ln z)^2 + \beta_3 \ln a \\
\ln \frac{k}{a} &= \theta_0 + \theta_1 \ln z + \theta_2 (\ln z)^2 + \theta_3 \ln a
\end{align*}
\]

\[
[\hat{\phi}_y, \hat{\phi}_k] := \arg \min_{\phi_y, \phi_k} \left( [\beta, \theta](\phi) - [\hat{\beta}, \hat{\theta}] \right) \Sigma \left( [\beta, \theta](\phi) - [\hat{\beta}, \hat{\theta}] \right)^T
\]

$\hat{\beta}, \hat{\theta}$: coefficients using empirical data

$\beta(\phi), \theta(\phi)$: coefficients using data simulated from model with $(\phi_y, \phi_k) = \phi$
\[
\phi_y = 0
\]

\[
\phi_y >> 0
\]
\[ \phi_y = 0 \]

\[ \phi_y >> 0 \]

Productivity vs. \[ \ln k/a \]

Productivity vs. \[ \ln y/k \]

Data vs. Model

Productivity vs. \[ \ln k/a \]

Productivity vs. \[ \ln y/k \]

Data vs. Model
## Fixed Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$ returns-to-scale</td>
<td>0.85</td>
<td>Midrigan &amp; Xu (2014)</td>
</tr>
<tr>
<td>$\alpha$ capital intensity</td>
<td>0.33</td>
<td>Midrigan &amp; Xu (2014)</td>
</tr>
<tr>
<td>$\rho$ productivity persistence</td>
<td>0.95</td>
<td>Moll (2014)</td>
</tr>
<tr>
<td>$\sigma$ productivity dispersion</td>
<td>0.627</td>
<td>90/10 ratio of productivity</td>
</tr>
</tbody>
</table>
Numerical routines

Solving the model:

- entrepreneur’s problem: value function iteration with linear interpolation
- stationary distribution: Young (2013) non-stochastic forward iteration method
- eqm price: bisection on $r$ and $w$

Finding the best parameters: brute force
for $(\phi_y, \phi_k) \in [0, 1]^2$ with 0.1 increments

- solve model
- simulate data (using the same $z$ history)
- run regressions
Distance on the \((\phi_y, \phi_k)\) grid

Distance between simulated and empirical coefficients

Best \(\phi_y = 0.6, \phi_k = 0.2\)
Distance on the \((\phi_y, \phi_k)\) Grid

Distance between simulated and empirical coefficients

Best \(\phi_y = 0.6, \phi_k=0.2\)

Best restricted \(\phi_y = 0, \phi_k=0.2\)
## Inference Results (OLS Weighting)

<table>
<thead>
<tr>
<th>Regression</th>
<th>Coefficient value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>best fit</td>
</tr>
<tr>
<td>$\phi_y = 0.6, \phi_k = 0.2$</td>
<td></td>
</tr>
<tr>
<td>Dep var $\ln \frac{k}{a}$</td>
<td></td>
</tr>
<tr>
<td>$\ln z$</td>
<td>2.489</td>
</tr>
<tr>
<td>$\ln a$</td>
<td>-0.379</td>
</tr>
<tr>
<td>$(\ln z)^2$</td>
<td>-0.789</td>
</tr>
<tr>
<td>Dep var $\ln \frac{y}{k}$</td>
<td></td>
</tr>
<tr>
<td>$\ln z$</td>
<td>1.456</td>
</tr>
<tr>
<td>$\ln a$</td>
<td>-0.275</td>
</tr>
<tr>
<td>$(\ln z)^2$</td>
<td>-0.216</td>
</tr>
<tr>
<td>Distance</td>
<td>0.368</td>
</tr>
</tbody>
</table>


Distance on the \((\phi_y, \phi_k)\) grid

Same D/Y as best
\(\phi_y = 0, \phi_k = 0.4\)

Best restricted
\(\phi_y = 0, \phi_k = 0.2\)

Best
\(\phi_y = 0.6, \phi_k = 0.2\)
## Inference Results (OLS Weighting)

<table>
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<td></td>
</tr>
<tr>
<td></td>
<td>$\phi_y = 0.6, \phi_k = 0.2$</td>
<td></td>
</tr>
<tr>
<td>Dep var $\ln \frac{k}{a}$</td>
<td>$\phi_y = 0$, same $\frac{D}{Y}$</td>
<td></td>
</tr>
<tr>
<td>$\ln z$</td>
<td>2.489</td>
<td>1.120 (0.050)</td>
</tr>
<tr>
<td>$\ln a$</td>
<td>-0.379</td>
<td>-0.512 (0.018)</td>
</tr>
<tr>
<td>$(\ln z)^2$</td>
<td>-0.789</td>
<td>-0.002 (0.013)</td>
</tr>
<tr>
<td>Dep var $\ln \frac{y}{k}$</td>
<td>$\phi_y = 0$, $\phi_k = 0.4$</td>
<td></td>
</tr>
<tr>
<td>$\ln z$</td>
<td>1.456</td>
<td>0.598 (0.035)</td>
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<tr>
<td>$\ln a$</td>
<td>-0.275</td>
<td>-0.206 (0.010)</td>
</tr>
<tr>
<td>$(\ln z)^2$</td>
<td>-0.216</td>
<td>-0.029 (0.008)</td>
</tr>
<tr>
<td>Distance</td>
<td>0.368</td>
<td>0.683</td>
</tr>
</tbody>
</table>
ROADMAP

- Model

- Quantification
  - Empirical patterns
  - Indirect inference

- Aggregate implications
Roadmap

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TFP loss due to financial frictions

First best TFP
\[ Z^{fb} := \left[ \mathbb{E}_z z^{\frac{1}{1-\eta}} \right]^{1-\eta} \]

TFP loss from financial frictions
\[ \text{loss} := \frac{Z^{fb} - Z}{Z}. \]
TFP loss (OLS weighting)

TFP loss from financial frictions

\[ \text{loss} := \frac{Z^{fb} - Z}{Z}. \]

Assuming \( \phi_y = 0 \) overstates loss due to financial frictions

\[ \begin{array}{|c|c|c|}
\hline
& \text{Best fit} & \text{Restricted} & \text{Restricted, same } \frac{D}{Y} \\
\hline
\phi_y = 0.6, \phi_y = 0.2 & 10.1\% & 15.3\% & 13.7\% \\
\hline
\end{array} \]

\textbf{Table:} TFP loss relative to the first best
Output loss (OLS weighting)

Output loss from financial frictions

\[
\text{loss} := \frac{Y^{fb} - Y}{Y}.
\]

Assuming \( \phi_y = 0 \) overstates loss due to financial frictions

<table>
<thead>
<tr>
<th></th>
<th>Best fit</th>
<th>Restricted</th>
<th>Restricted, same ( \frac{D}{Y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_y )</td>
<td>( \phi_y = 0.6, \phi_y = 0.2 )</td>
<td>( \phi_y = 0, \phi_k = 0.2 )</td>
<td>( \phi_y = 0, \phi_k = 0.4 )</td>
</tr>
<tr>
<td>( \phi_k )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.0%</td>
<td>42.7%</td>
<td>35.3 %</td>
</tr>
</tbody>
</table>

**Table:** TFP loss relative to the first best
## Robustness Check (Equal Weighting)

<table>
<thead>
<tr>
<th>Regression</th>
<th>Coefficient value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>best fit</td>
<td>( \phi_y = 0 )</td>
</tr>
<tr>
<td>( \text{Dep var } \ln \frac{k}{a} )</td>
<td>( \phi_y = 0.5, \phi_k = 0.2 )</td>
<td>( \phi_y = 0, \phi_k = 0.3 )</td>
</tr>
<tr>
<td>( \ln z )</td>
<td>2.297</td>
<td>1.710</td>
</tr>
<tr>
<td>( \ln a )</td>
<td>-0.351</td>
<td>-0.318</td>
</tr>
<tr>
<td>( (\ln z)^2 )</td>
<td>-0.852</td>
<td>-1.253</td>
</tr>
<tr>
<td>( \text{Dep var } \ln \frac{y}{k} )</td>
<td>( \phi_y = 0.5, \phi_k = 0.2 )</td>
<td>( \phi_y = 0, \phi_k = 0.3 )</td>
</tr>
<tr>
<td>( \ln z )</td>
<td>1.532</td>
<td>1.727</td>
</tr>
<tr>
<td>( \ln a )</td>
<td>-0.226</td>
<td>-0.238</td>
</tr>
<tr>
<td>( (\ln z)^2 )</td>
<td>0.297</td>
<td>0.437</td>
</tr>
<tr>
<td>Distance</td>
<td>0.516</td>
<td>0.574</td>
</tr>
</tbody>
</table>
Robustness check (equal weighting)

Assuming $\phi_y = 0$ overstates loss due to financial frictions.

<table>
<thead>
<tr>
<th>$\phi_y = 0.5$, $\phi_k = 0.2$</th>
<th>$\phi_y = 0$, $\phi_k = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.7%</td>
<td>14.6%</td>
</tr>
</tbody>
</table>

Table: TFP loss relative to the first best
CONCLUSION

For young and unlisted Japanese firms
  ▪ leverage increases with productivity
  ▪ output-capital ratio increases with productivity

Pattern is consistent with a model of leverage capacity increasing with productivity

Accounting for this empirical pattern matters for understanding TFP loss due to financial frictions.