Screening and Adverse Selection in Frictional Markets

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Introduction

Many markets feature **adverse selection and imperfect competition**

- Examples: insurance, loans, financial securities
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- Large empirical literature (and some theory)
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But many important questions

- Recent push to make these markets more competitive, transparent
- Is this a good idea?
This Paper

A tractable model of adverse selection, screening and imperfect comp.

1. Complete characterization of the unique equilibrium
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2. Explore positive predictions for distribution of contracts
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2. Explore positive predictions for distribution of contracts

3. Policy experiments: changes in competition, transparency
Sketch of Model: Key Ingredients

• Adverse Selection: sellers have private info about quality

• A fraction $\mu h$ have quality $h$, the rest quality $\ell$

• Screening: Buyers offer general menus of non-linear contracts

• Price-quantity pairs: induce sellers to self-select

• Imperfect Comp: sellers receive either 1 or 2 offers (à la Burdett-Judd)

• Buyer competing with another with prob $\pi$, otherwise monopsonist.

• Contract offered before buyers know
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What We Know (Equilibrium)

\[ \pi \]

\[ \mu_h \]

\[ 0 \]

\[ (\text{Monopsony}) \]

\[ \bar{\mu}_h \]

\[ 1 \]

\[ (\text{Perfect Comp}) \]
Perfect competition and “severe adverse selection” \(\Rightarrow\) least-cost separation.
Perfect competition and “mild adverse selection” \( \Rightarrow \) Mixed Strategy Eq.
What We Know (Equilibrium)

Monopsony and “severe adverse selection” $\Rightarrow$ No Trade with High Type
What We Know (Equilibrium)

Monopsony and “mild adverse selection” ⇒ Full Trade

Pool High and Low Quality Sellers (Stiglitz ‘77)

No Trade With High Quality Seller (Stiglitz ‘77)

Only Mixed Strategy Equilibria (Rosenthal-Weiss ‘84)

Least Cost Separating Outcome (Rothschild-Stiglitz ‘76)

Monopsony and “mild adverse selection” ⇒ Full Trade
Objective

Obj: Characterize eqm for any degree of adverse selection and imperfect comp.
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Financial and Insurance markets typically characterized by imperfect comp.
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Financial and Insurance markets typically characterized by imperfect comp.

What are the implications of imperfect comp. for....

- Terms of trade
- Welfare
- Policy
Summary of Findings

Methodology

• New techniques to characterize unique eqm for all \((\mu_h, \pi) \in [0, 1]^2\)
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Methodology

- New techniques to characterize unique eqm for all \((\mu_h, \pi) \in [0, 1]^2\)
- Establish important (and general!) property of all equilibria:
  - *Strictly rank preserving*: offers for \(\ell\) and \(h\) ranked exactly the same
    - No specialization
- Positive Implications
  - Equilibrium can be pooling, separating, or mix
  - Separation when adverse selection severe, trading frictions mild
  - Pooling when adverse selection mild, trading frictions severe
- Normative Implications
  - Adverse selection severe: interior \(\pi\) maximizes surplus from trade
  - Adverse selection mild: welfare unambiguously decreasing in \(\pi\)
  - Increasing transparency/relaxing info frictions can ↑ or ↓ welfare
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Related Literature

**Empirical**

- Chiappori and Salanie (2000); Ivashina (2009); Einav et al. (2010); Einav et al. (2012)

**Adverse Selection and Screening**

- Rothschild and Stiglitz (1976); Dasgupta and Maskin (1986); Rosenthal and Weiss (1984); Mirrlees (1971); Stiglitz (1977); Maskin and Riley (1984); Guerrieri, Shimer and Wright (2010); Many, many others

**Imperfect Competition and Selection**

- Search Frictions: Burdett and Judd (1983); Garrett, Gomes, and Maestri (2014)

ENVIRONMENT
Model Environment

Large number of buyers and sellers
Model Environment

Large number of buyers and sellers

- Each Seller endowed with 1 divisible asset
  - Seller values asset at rate $c_i$
  - Two types of sellers $i \in \{l, h\}$ with prob. $\mu_i$

- Buyer values type $i$ asset at rate $v_i$
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- If $x$ units sold for transfer $t$, payoffs are
  - Seller: $t + (1 - x)c_i$
  - Buyer: $xv_i - t$

Assumptions:
- Gains to trade: $v_i > c_i$
- Lemons Assumption: $v_l < c_h$
- Adverse Selection: Only sellers know asset quality
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Screening

- Buyers post arbitrary menus of exclusive contracts
- Screening menus intended to induce self-selection
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Search frictions

- Each seller receives 1 offer w.p. $1 - \pi$ and both w.p. $\pi$
  - Refer to seller with 1 offer as Captive
  - Refer to seller with 2 offers as non-Captive
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Stylized Model of Trade

- best examples: corporate loans market; securitization (maybe)
- other examples: information-based trading; insurance
Strategies

- Each buyer offers arbitrary menu of contracts \( \{(x_n, t_n)_{n \in \mathcal{N}}\} \)
- Captive seller’s choice: best \((x_n, t_n)\) from one buyer
- Non-captive seller’s choice: best \((x_n, t_n)\) among both buyers
Strategies

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Revelation Principle

sufficient to consider

- menus with two contracts \( z \equiv \{(x_l, t_l), (x_h, t_h)\} \)

\[(IC_j) : t_j + c_j(1 - x_j) \geq t_{-j} + c_j(1 - x_{-j}) \quad j \in \{h, l\}\]

- seller \(j\): chooses contract \(j\) from available the set of menus available
Suppose $\pi \in (0, 1)$: no symmetric pure strategy equilibrium exists
  - buyers can guarantee positive profits: trade only with captive types
  - in a pure strategy equilibrium: have to share non-captive types
Equilibrium Price Dispersion

- Suppose $\pi \in (0, 1)$: no symmetric pure strategy equilibrium exists
  - buyers can guarantee positive profits: trade only with captive types
  - in a pure strategy equilibrium: have to share non-captive types
    There is always an incentive to undercut

- Only mixed strategy equilibria possible
  $\Rightarrow$ equilibrium features price dispersion
  $\Rightarrow$ equilibrium described by buyers’ distribution over menus
Equilibrium definition

A symmetric equilibrium is a distribution \( \Phi(z) \) such that almost all \( z \) satisfy,

1. **Incentive compatibility:**

\[
 t_j + c_j(1 - x_j) \geq t_{-j} + c_j(1 - x_{-j}) \quad j \in \{h, l\}
\]

2. **Seller optimality:**

\( \chi_i(z, z') \) maximizes her utility

3. **Buyer optimality:** for each \( z \in \text{Supp}(\Phi) \)

\[
 z \in \arg \max_z \sum_{i \in \{l, h\}} \mu_i(v_i x_i - t_i) \left[ 1 - \pi + \pi \int_{z'} \chi_i(z, z') \Phi(dz') \right] \\
\]
Characterization

Equilibrium described by non-degenerate distribution in 4 dimensions
Characterization

Equilibrium described by non-degenerate distribution in 4 dimensions

Proceed in 4 steps

1. Show that menus can be summarized by a pair of utilities \((u_h, u_l)\)
   - Reduces dimensionality of problem to distribution in 2 dimensions

2. Show there is a 1-1 mapping between \(u_l\) and \(u_h\)
   - Reduces problem to distribution in 1 dimension + a monotonic function

3. Construct Equilibrium

4. Show that constructed equilibrium is unique
Result (Dasgupta and Maskin (1986))

In all menus offered in equilibrium,

- the low types trades everything: \( x_l = 1 \)
- IC\(_l\) binds: \( t_l = t_h + c_l(1 - x_h) \)
A utility representation

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Result

Equilibrium menus can be represented by \((u_h, u_l)\) with corresponding allocations

\[
\begin{align*}
  t_l &= u_l \\
  x_h &= 1 - \frac{u_h - u_l}{c_h - c_l} \\
  t_h &= \frac{u_l c_h - u_h c_l}{c_h - c_l}
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Since we must have \(0 \leq x_h \leq 1\),

\[
  c_h - c_l \geq u_h - u_l \geq 0
\]
A utility representation

Marginal distributions

\[ F_j(u) = \int_{z'} 1 \left[ t'_j + c_j (1 - x'_j) \leq u_j \right] \, d\Phi(z') \quad j \in \{h, l\} \]
A utility representation

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Then, each buyer solves

\[ \Pi(u_h, u_l) = \max_{u_l \geq c_l, \ u_h \geq c_h} \sum_{j \in \{h, l\}} \mu_j \left[ 1 - \pi + \pi F_j(u_j) \right] \Pi_j(u_h, u_l) \]

\[ \text{s. t.} \quad c_h - c_l \geq u_h - u_l \geq 0 \]
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with \[ \Pi_l(u_h, u_l) \equiv v_l x_l - t_l = v_l - u_l \]

\[ \Pi_h(u_h, u_l) \equiv v_h x_h - t_h = v_h - u_h \frac{v_h - c_l}{c_h - c_l} + u_l \frac{v_h - c_h}{c_h - c_l} \]
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Need to characterize the two linked distributions \( F_l \) and \( F_h \)!
Further Simplifying the Characterization

Result

$F_l$ and $F_h$ have connected support and are continuous.

- Except for a knife-edge case (see paper)
- Proof more involved than standard case because of interdependencies
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Result

The profit function $\Pi(u_h, u_l)$ is strictly supermodular.

- Intuition: $u_l \uparrow \Rightarrow \Pi_h \uparrow \Rightarrow$ stronger incentives to attract high types
- $\Rightarrow U_h(u_l) \equiv \arg\max_{u_h} \Pi(u_h, u_l)$ is weakly increasing
Strict Rank Preserving

Theorem

\[ U_h(u_l) \text{ is a strictly increasing function.} \]
**Strict Rank Preserving**

**Theorem**

$U_h(u_l)$ is a strictly increasing function.

**Idea of Proof**

- $U_h(u_l)$ increasing due to super-modularity of profit function
- $F_l$ and $F_h$ have no holes or mass points imply $U_h$ is strictly increasing and not a correspondence
Strict Rank Preserving

**Theorem**

\[ U_h(u_l) \text{ is a strictly increasing function.} \]

Implications for Characterization

- Rank ordering of equilibrium menus identical across types
- Menus attract same fraction of both types \( F_l(u_l) = F_h(U_h(u_l)) \)
- Greatly simplifies the analysis: only have to find \( F_l(u_l) \) and \( U_h(u_l) \)

Broader Implications

- Buyers do not specialize or attract only a subset of types
- Terms of trade offered to both types are positively correlated
- Robust to any number of types
- Relies only on utility representation and ability to show distributions are well behaved
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**Theorem**

$U_h(u_l)$ is a *strictly increasing function*.

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CONSTRUCTING EQUILIBRIA
Equilibria: The two limit cases

Monopsony: \( \pi = 0 \)

Bertrand: \( \pi = 1 \)
Equilibria: The two limit cases

Monopsony: $\pi = 0$

- $\mu_h < \bar{\mu}_h \Rightarrow$ Sep. with $x_h = 0$ and $\Pi_l > \Pi_h = 0$
  - No Cross-subsidization
- $\mu_h \geq \bar{\mu}_h \Rightarrow$ Pooling with $x_h = x_l = 1$ and $\Pi_h > 0 > \Pi_l$
  - Cross-subsidization

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Equilibria: The two limit cases

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  - Cross-subsidization

Intuition: Higher $\mu_h \Rightarrow$ Relaxing $IC^l$ more attractive
Types of equilibria in the middle

No cross-subsidization

Cross-subsidization

\[ \mu_h \times \Pi_h > \Pi_l > 0 \]

\[ \mu_h \times (\Pi_l, \Pi_h) \geq 0 \]

\[ u_h(\mu_l) \neq u_l^{2/4} \]
Types of equilibria in the middle

High $\mu_h$
- $\Pi_h > 0 > \Pi_l$
- All separating, all pooling or a mix

Low $\mu_h$
- $\Pi_l, \Pi_h \geq 0$
- All separating, $U_h(u_l) \neq u_l$
No cross-subsidization: Characterization

Focus on separating equilibrium in no-cross subsidization region

Recall problem of a buyer:

\[
\Pi(u_h, u_l) = \max_{u_l \geq c_l, \ u_h \geq c_h} \sum_{j \in \{l, h\}} \mu_j [1 - \pi + \pi F_j(u_j)] \Pi_j(u_h, u_l)
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s. t. \quad c_h - c_l \geq u_h - u_l \geq 0

- In separating equilibrium we construct, \( c_h - c_l > u_h - u_l > 0 \)
- Sufficient to ensure local deviations unprofitable
No cross-subsidization: Characterization

Marginal benefits vs costs of increasing $u_l$

\[
\frac{\pi f_i (u_l) \Pi_l}{1 - \pi + \pi F_l (u_l)} + \frac{\mu_h}{1 - \mu_h} \frac{v_h - c_h}{c_h - c_l} = \frac{1}{MC}
\]

MB of more low types

MB of relaxing $IC_l$
No cross-subsidization: Characterization

Marginal benefits vs costs of increasing $u_l$

$$\frac{\pi f_l(u_l) \Pi_l}{1 - \pi + \pi F_l(u_l)} + \frac{\mu_h \nu_h - c_h}{1 - \mu_h} \frac{c_h - c_l}{c_l} = \frac{1}{MC}$$

MB of more low types  
MB of relaxing $IC_l$

Boundary conditions

$$F_l(c_l) = 0 \quad F_l(\bar{u}_l) = 1 \quad \rightarrow \quad F_l(u_l)$$

Equal profit condition

$$[1 - \pi + \pi F_l(u_l)] \Pi(\bar{U}_h, u_l) = \bar{\Pi} \quad \rightarrow \quad U_h(u_l)$$
No cross-subsidization: Characterization

Marginal benefits vs costs of increasing $u_l$

$$\frac{\pi f_l(u_l) \Pi_l}{1 - \pi + \pi F_l(u_l)} + \frac{\mu_h v_h - c_h}{1 - \mu_h c_h - c_l} = \frac{1}{MC}$$

MB of more low types

MB of relaxing $IC_l$

Boundary conditions

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Equal profit condition

$$[1 - \pi + \pi F_l(u_l)] \Pi(U_h, u_l) = \bar{\Pi} \quad \rightarrow \quad U_h(u_l)$$

Pursue similar construction in other regions of parameter space
Equilibrium Regions in the Middle

- No cross-subsidization
- Separation
- Pooling
- Mix

More Competition implies less pooling

- Gains to cream-skimming increase in $\pi$
- Milder Adverse Selection (higher $\mu_h$) implies more pooling
- Increased incentives to trade high volume
- Increased cost of cream-skimming

Price Dispersion Theorem

For every $(\pi, \mu_h)$ there is a unique equilibrium.
Equilibrium Regions in the Middle

More Competition implies less pooling
- Gains to cream-skimming increase in $\pi$

Milder Adverse Selection (higher $\mu_h$) implies more pooling
- increased incentives to trade high volume
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Theorem

*For every $(\pi, \mu_h)$ there is a unique equilibrium.*
EQUILIBRIUM IMPLICATIONS
Positive and Normative Implications

Is improving competition desirable for volume or welfare?

- For high $\mu_h$, monopsony dominates perfect competition
- For low $\mu_h$, perfect competition dominates monopsony
- Will show: for low $\mu_h$, welfare maximized at interior $\pi$
Positive and Normative Implications

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Is increasing transparency desirable?

- Allowing insurers, loan officers, dealers to discriminate on observables?
- Interpret increased transparency as increased spread in $\mu_h$
- Desirability depends on curvature of welfare function with respect to $\mu_h$
- Will show: Concavity/Convexity of welfare function depends on $\pi, \mu_h$
EQUILIBRIUM IMPLICATIONS: COMPETITION
Assume $\mu_h$ in no cross-subsidization region
Competition with No Cross-Subsidization

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Equilibrium Distribution and $U_h(u_l)$ for $\pi = 0.2$

Shaded Region indicates support of $F_l$
Competition with No Cross-Subsidization

Assume $\mu_h$ in no cross-subsidization region

Equilibrium Distribution and $U_h(u_l)$ for $\pi = 0.5$

- Shaded Region indicates support of $F_l$

- Increase in $\pi$ increases $F_l$ in sense of FOSD
Competition with No Cross-Subsidization

Assume $\mu_h$ in no cross-subsidization region

Equilibrium Distribution and $U_h(u_l)$ for $\pi = 0.9$

Shaded Region indicates support of $F_l$

- Increase in $\pi$ increases $F_l$ in sense of FOSD
- Driven by increased competition for (abundant) low-quality sellers
Competition with No Cross-Subsidization

How is trade volume related to $U_h$?

$$x_h(u_l) = 1 - \frac{U_h(u_l) - u_l}{c_h - c_l}$$

$$x_h'(u_l) > 0 \iff U_h'(u_l) > 1$$
Competition with No Cross-Subsidization

Equilibrium Objects for $\pi = 0.2$
Competition with No Cross-Subsidization

Equilibrium Objects for $\pi = 0.5$

- From low $\pi$, increase in $\pi$ increases volume
Competition with No Cross-Subsidization

Equilibrium Objects for $\pi = 0.9$

- From moderate $\pi$, increase in $\pi$ decreases volume
Competition and Welfare

When no cross-subsidization

\[ W(\mu_h, \pi) = (1 - \mu_h)(v_l - c_l) + \mu_h(v_h - c_h) \int x_h(u_l)\,dF(u_l) \]

Why is welfare decreasing?

- \( \mu_h \) implies few high types
- Competition less fierce for high types
- Demand from high types relatively inelastic
- Equal profits \( \Rightarrow \) greater dispersion in prices
- Implies \( U'_h(u_l) > 1 \)

Welfare maximized for interior
Competition and Welfare

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Welfare maximized for interior \(\pi\)
Competition and Welfare

When no cross-subsidization

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Welfare maximized for interior \( \pi \)

With Cross-Subsidization, welfare (weakly) maximized in monopsony outcome
- Full trade \( \Rightarrow \) all gains to trade exhausted
EQUILIBRIUM IMPLICATIONS: TRANSPARENCY
Desirability of Transparency

Do the following policies improve welfare?

- Allowing insurance providers to discriminate based on observables
- Introducing credit scores in loan markets
- Requiring OTC market participants to disclose trades
Desirability of Transparency

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In model, interpret increased transparency as mean-preserving spread of $\mu_h$

- Each seller has individual $\mu'_h$; Buyers know distribution over $\mu'_h$
- Buyers restricted to offering contracts associated with $E[\mu'_h]$
- Under transparency, buyers allowed to offer $\mu_h$-specific menus
- Need to compare $E[W(\mu'_h, \pi)]$ to $W(E[\mu'_h], \pi)$

Is Transparency Desirable? Answer: Depends on $\pi$!

- $W$ is linear when $\pi = 0$ and $\pi = 1$ ⇒ no effect on welfare
- $W$ is concave when $\pi$ is high ⇒ bad for welfare
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Desirability of Transparency: The two limit cases

Monopsony: \( \pi = 0 \)

Bertrand: \( \pi = 1 \)
Desirability of Transparency: The two limit cases

Monopsony: $\pi = 0$

- $\mu_h < \bar{\mu}_h \Rightarrow x_h = 0$ so that
  
  \[ W(\mu_h) = (1 - \mu_h) v_l + \mu_h c_h \]

- $\mu_h > \bar{\mu}_h \Rightarrow x_h = 1$ so that
  
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- Welfare is linear in $\mu_h$

Bertrand: $\pi = 1$

- $\mu_h < \bar{\mu}_h \Rightarrow x_h$ independent of $\mu_h$
- Implies welfare is linear in $\mu_h$
Desirability of Transparency: The two limit cases

Monopsony: \( \pi = 0 \)

- \( \mu_h < \bar{\mu}_h \Rightarrow x_h = 0 \) so that
  \[
  W(\mu_h) = (1 - \mu_h)v_l + \mu_h c_h
  \]

- \( \mu_h > \bar{\mu}_h \Rightarrow x_h = 1 \) so that
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Bertrand: \( \pi = 1 \)

- \( \mu_h < \bar{\mu}_h \Rightarrow x_h \) independent of \( \mu_h \)

- Implies welfare is linear in \( \mu_h \)

In these cases, welfare is linear in \( \mu_h \) so that mean-preserving spread (locally) has no impact on welfare
Desirability of Transparency: The general cases

- With cross-subsidization, welfare is concave
  \( \Rightarrow \) increases in transparency harm welfare

- Without cross-subsidization, welfare is concave only for high \( \pi \)
  \( \Rightarrow \) increases in transparency harm welfare when markets competitive
Conclusion

Methodological contribution

- Imperfect competition and adverse selection with optimal contracts
- Rich predictions for the distribution of observed trades

Substantive insights

- Depending on parameters, pooling and/or separating menus in equilibrium
- Competition, transparency can be bad for welfare

Work in progress

- Generalize to $N$ types, curved utility
- Non-exclusive trading
No cross-subsidization: Price vs quantity (conditional)

\[
\pi = 0.2
\]

\[
\pi = 0.5
\]

\[
\pi = 0.95
\]

Correlation \( < 0 \) for suff. high \( \pi \)

A strategy to infer competitiveness?