A Theory of Public Debt Overhang

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1 Introduction

2 Benchmark Model
   • Model without public debt overhang
   • Model with public debt overhang

3 Modified model with multiple equilibria

4 Conclusion
Reinhart, Reinhart, and Rogoff (2012)

- Decade-long stagnation in debt-ridden economy.
- Growth rate declines by 1 % as public debt exceeds 90 % of GDP.
- In some cases, low growth is associated with low interest rates (eg., Japan).
Higher REAL INTEREST RATES

Higher GROWTH

Belgium, 1920–1926
Netherlands, 1932–1954

Interest rates about the same

UK, 1830–1868

Lower GROWTH

Lower REAL INTEREST RATES

Belgium, 1982–2005
Canada, 1992–1999
France, 1880–1905
Greece, 1848–1883
Greece, 1887–1913
Greece, 1928–1939
Greece, 1993–2011
Ireland, 1983–1993
Italy, 1881–1904
Italy, 1917–1936
Italy, 1940–1944
Italy, 1988–2011
Netherlands, 1816–1862
Spain, 1868–1882
Spain, 1896–1909

Australia, 1945–1950 (−0.1/−7.3)
U.S., 1944–1949
France, 1920–1945
Japan, 1995–2012

Interest rates about the same

Canada, 1944–1950
Netherlands, 1886–1898
New Zealand, 1881–1951
UK, 1917–1964

Source: Reinhart, Reinhart, Rogoff(2012)
What we do

we propose a simple model in which

unsustainable debt undermines credibility of government’s commitment,

commitment problem discourages adoption of new technology, leading to low growth and a low interest rate.

fiscal consolidation restores sustainability of public debt,

and leads to high growth and a high interest rate by restoring credibility of government commitments (basic model).

but cannot restore credibility of government commitments if it comes too late (second model).
1 Introduction

Related literature

- **Empirics**
  - Reinhart, Reinhart and Rogoff (2012)
    - 26 episodes of advanced economies
  - Checherita-Westphal and Rother (2012)
    - 12 euro countries
  - Baum, Checherita-Westphal and Rother (2013)

- **Crowding out**
Non-Keynesian effect (Giavazzi and Pagano 1990, Bertola and Drazen 1993, Perotti 1999)

- expansionary fiscal policy leads to low consumption.
- inefficiency is caused by expectations of one-time tax distortion in the future.
- it is not consistent with a decade of low growth.
Growth theory

- Diamond (1960): public debt enhances growth in a neoclassical model.

Political economy (Acemoglu and Robinson 2005, Acemoglu, 2009)

- commitment problem due to political conflict lowers investment.
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Benchmark model without public debt overhang

- Continuous time AK model. Capital does not depreciate.
- A consumer, a government, and $N$ firms. ($N \gg 1$)
- Firms are owned by the consumer.
- The consumer’s utility:
  \[ \int_{0}^{\infty} e^{-\rho t} \ln c_t, \quad \text{where} \quad 0 < \rho. \]
- The incumbent government wants to maximize the length of its tenure $T$. If $T = \infty$ the government maximizes welfare of consumers.
- A firm borrows $\frac{k_t}{N}$ from the consumer and produce the consumption good in period $t$. 

Benchmark model without public debt overhang

- Two technologies, A and B

0 < B < A.

- Technology A: a firm with $k_t$ can produce $A k_t$ in period $t$. The government can impose the output tax $\tau_{k_t}A k_t$, where $0 \leq \tau_{k_t} \leq 1$.
- Technology B: a firm with $k_t$ can produce $B k_t$ in period $t$. The government cannot impose the output tax on $B k_t$.

Without public debt overhang, the government has no incentive to impose output tax.

Thus firms choose technology A.
Benchmark model without public debt overhang

- consumer’s problem

\[
\max_{c_t, k_t} \int_0^\infty e^{-\rho t} \ln c_t, \\
\text{s.t. } c_t + \dot{k}_t = R_t k_t + X_t.
\]

- a firm’s problem

\[
\max_y \ y - R_t \frac{k_t}{N}, \\
\text{s.t. } y \in \left\{ \frac{k_t}{A N}, \frac{k_t}{B N} \right\}
\]
Benchmark model without public debt overhang

- government’s problem
  - tenure of incumbent government: \( T = \infty \).
  - tax policy does not change the length of tenure.
  - government has no incentive to set \( \tau_{kt} \neq 0 \).
    - It is credible that \( \tau_{kt} = 0 \).
      - \( \Rightarrow \) Firms choose technology A.
Equilibrium

- Firms choose technology A.
- Government chooses $\tau_{kt} = 0$
- standard AK outcome:

\[
\begin{align*}
R_t &= A, \\
X_t &= 0, \\
k_t &= k_0 e^{(A-\rho)t}, \\
c_t &= \rho k_t, \\
\zeta_t &= \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = A - \rho.
\end{align*}
\]
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Public debt overhang

- Outstanding government debt \( b_0 \).
- No tax is imposed in \( t = 0 \).
- For \( 0 \leq t (< T) \), consumer’s problem is

\[
\max_{c_t, k_t, b_t} \int_0^\infty e^{-\rho t} \ln c_t, \\
s.t. \quad c_t + \dot{k}_t + \dot{b}_t = R_t k_t + r_t b_t + X_t.
\]

- Debt evolves by

\[
\dot{b}_t = r_t b_t.
\]
Two taxes

- Lump-sum tax and output tax
  - Lump-sum tax, $\tau_{t+1}$, on the consumer.
    \[ \tau_t \in [0, \tau]. \]
    - The government loses power if the lump-sum tax is imposed.
    - If the lump-sum tax is imposed at $T$, the tenure of incumbent government is terminated at $T$.
  - Output tax, $\tau_k A_t$, on the firms.
    \[ \tau_{kt} \in [0, 1]. \]
    - Government can impose output tax without any cost.
    - Output by technology A is taxable, while output by technology B is not taxable.
    - Output tax can be imposed after observing production.
2 Benchmark Model
2.2 Model with public debt overhang

Transversality condition

- Government can continue until $T$, where

$$T = \arg \max_T b_T,$$

s.t. \[ \dot{b}_t = r_L b_t, \]

$$r_H b_T \leq \tau,$$

where $r_H = A$ and $r_L = B.$
Lemma. Given that the lump-sum tax \( \tau_t = \tau \) is imposed in period \( T \), the TVC from \( T \) on is satisfied iff

\[
r_H b_T \leq \tau.
\]

- Debt evolves by \( \dot{b}_t = rb_t - \tau_t \) for \( t \geq T \), where \( \tau_t = \tau \) if \( b_t > 0 \) and \( \tau_t = 0 \) if \( b_t = 0 \).
- Thus \( b_{t+T} = \max \{ 0, \frac{\tau}{r} + (b_T - \frac{\tau}{r}) e^{rt} \} \) if \( b_T \leq \frac{\tau}{r} \);
  and \( b_{t+T} = \frac{\tau}{r} + (b_T - \frac{\tau}{r}) e^{rt} \) if \( b_T > \frac{\tau}{r} \).
- TVC is \( \lim_{t \to \infty} e^{-rt} b_t = 0 \), which is equivalent to \( b_T \leq \frac{\tau}{r} \).
- can show \( r \neq r_L \) by contradiction: if \( r = r_L \) and TVC is satisfied, the government has no incentive to impose output tax; then firms choose technology A, leading the interest rate to \( r = r_H \).
- The equilibrium interest rate is \( r_t = r_H \) for \( t \geq T \).
Define $r_H b_T = \tau$. Incumbent government cannot stay for $t > T$.

- Consumer does not buy $b_t$ unless $\tau$ is introduced.
- If $\tau$ is introduced at $T$, incumbent loses power.
- If $\tau$ is not introduced at $T$, incumbent loses power because default on government bond occurs at $T$.

Consumers accept to buy $b_t$ for $t > T$ iff $\tau$ is introduced and TVC is satisfied.
Firms choose technology B due to political economy distortion (Acemoglu 2009):

- At every period $t$, government decides whether to introduce output tax after observing firms’ choice of technology (A or B).
- Government cannot precommit not to introduce output tax.
- Equilibrium is Markov Perfect Equilibrium. Punishment strategy is excluded.
Lack of Commitment

- If firms choose technology A, government imposes output tax and sets $\tau_{kt} = 1$. (government takes all output.)
- Government can extend the tenure $T$ by imposing output tax if technology A is adopted.
  - In continuous time model, tax revenue at $t$ is infinitesimally small and does not affect the length of tenure $T$?
    We can justify the above statement by the following argument:
    - We assume that if firms choose technology A at $t$ they cannot change technology until $t + \Delta$, where $\Delta \ll 1$ is a very short time interval.
    - We assume that capital stock allocated to one firm cannot be reallocated to other firms; in other words, capital stock of each firm $k_t$ must satisfy $k_s \geq k_t$ for all $s \geq t$.
    - Suppose that a firm that owns $\frac{k_t}{N}$ chooses technology A at $t$.
    - At $t' = t + \Delta$, the government debt $b_{t'}$ becomes
      \[
      b_{t'} \approx b_t + \left( r_t b_t - A \frac{k_t}{N} \right) \Delta 
      < b_t + r_t b_t \Delta.
      \]
  - Since the amount of debt is lower by approximately $A \frac{k_t}{N} \Delta$ with output tax than without it, the tenure can be extended by $O(\Delta)$ by imposing output tax if firms adopt technology A.

- Anticipating this, all firms choose technology B for $0 \leq t < T$. 
Equilibrium with Public Debt Overhang

- $T = \arg \max_t b_t$ subject to $\dot{b}_t = r_L b_t$ and $r_H b_t \leq \tau$.

- Firms choose technology B for $0 \leq t < T$.

\[
\begin{align*}
    r_t &= \begin{cases} 
        r_L = B, & \text{for } 0 \leq t < T, \\
        r_H = A, & \text{for } t \geq T,
    \end{cases} \\
    b_t &= \begin{cases} 
        b_0 e^{r_L t}, & \text{for } 0 \leq t < T, \\
        b_T, & \text{for } t \geq T,
    \end{cases} \\
    k_t &= \begin{cases} 
        k_0 e^{(r_L-\rho)t}, & \text{for } 0 \leq t < T, \\
        k_T e^{(r_H-\rho)(t-T)}, & \text{for } t \geq T,
    \end{cases} \\
    c_t &= \rho k_t.
\end{align*}
\]
Equilibrium with Public Debt Overhang

- Growth rate is low before fiscal consolidation, while it is high after fiscal consolidation.
  \[
  \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_L - \rho, \quad \text{for } 0 \leq t < T,
  \]
  \[
  \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_H - \rho, \quad \text{for } t \geq T,
  \]

- Interest rate is low before fiscal consolidation, while it is high after fiscal consolidation.

- Output tax is not imposed in equilibrium.
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Modified model – No credible fiscal policy

- AK model
- Output tax $\tau_k$ is already imposed.
- Two technology. A and B

\[ A - \gamma < B < A \]

- Technology A: Firm produces $Ak_t$. Firm needs to pay output tax $\tau_kAk_t$ and political rent $\gamma k_t$ to consumer.
  - $\gamma k_t$ represents rent associated with education of workers and/or investment in infrastructure for new technology A.
- Technology B: Firm produces $Bk_t$. Firm needs to pay output tax $\tau_kBk_t$. Firm with technology B does not have to pay political rent.
Modified model – Description of fiscal policy

- Fiscal policy $g_t$: government pays $g_t k_t$ to firms that use technology A.
  - $g_t k_t$ represents redistribution associated with education and/or public investment in infrastructure for new technology.
- Profit of firm that uses technology A
  
  $$[(1 - \tau_k)A - \gamma + g_t]k_t$$

- Profit of firm that uses technology B
  
  $$(1 - \tau_k)Bk_t$$

- Firm chooses A if
  
  $$g_t \geq \gamma - (1 - \tau_k)(A - B).$$

- Tax revenue decrease if government pays $g_t$:
  
  $$\tau_k Ak_t - g_t \leq \tau_k B.$$
Large initial value $d_0 > \frac{\tau_k Bk_0}{\rho} > 0$.

Debt evolves by $\dot{b}_t = r_t b_t - \tau_k Bk_t$. Therefore,

$$b_t = \frac{\tau_k Bk_0}{\rho} e^{(r_L - \rho)t} + \left(b_0 - \frac{\tau_k Bk_0}{\rho}\right) e^{rLt}.$$

Debt diverges at the rate $r_L$.

Lump-sum tax, $\tau$

- Government can impose lump-sum tax $\tau$ on consumer to restore TVC.
- If lump-sum tax is introduced at $T$, the incumbent’s tenure is terminated at $T$. 
Modified model – Lack of commitment

- Government decides whether to pay $g_kt$ after observing choice of technology A or B.
- If firms choose technology A,
  - government revenue increases: $\tau_k A k_t \ (> \tau_k B k_t)$ if the government does not pay $g_t k_t$.
  - government revenue decreases: $(\tau_k A - g_t) k_t \ (< \tau_k B k_t)$ if it pays $g_t k_t$, where $g_t > \gamma - (1 - \tau_k)(A - B)$.
government wants to extend its tenure, $T$, by increasing revenue.

government does not pay $g_t$ if firms choose A.

- In continuous time model, tax revenue at $t$ is infinitesimally small and does not affect the length of tenure $T$. We can justify the above statement by the following argument:
  - We assume that if firms choose technology A at $t$ they cannot change technology until $t + \Delta$, where $\Delta \ll 1$ is a very short time interval.
  - Observing firms’ choice of technology at $t$, government decides whether to pay $g_k s$ for $s \geq t$.
  - The amount of debt is lower by approximately $g_k t \Delta$ when government pays subsidy than when it does not.
  - The tenure is strictly longer by $O(\Delta)$ when government does not pays $g_k t$ than when it does, whatever technology firms choose.

- Government does not pay $g_k t$.

anticipating this, all firms choose B.
3 Modified model with multiple equilibria

**Modified model – Optimization problems**

- **Consumer’s problem**

  \[
  \max \int_0^\infty e^{-\rho t} \ln c_t, \\
  \text{s.t.} \quad c_t + \dot{k}_t + \dot{b}_t = R_t k_t + r_t b_t + X_t.
  \]

- **Firm’s problem:** \[\max \left[ \max \{ (1 - \tau_k)A - \gamma + g_t, (1 - \tau_k)B \} - R_t, 0 \right].\] Firms choose A iff \[g_t \geq \gamma - (1 - \tau_k)(A - B).\]

- **Government’s problem**

  \[
  \max_{g_t} T, \\
  \begin{cases}
  \dot{b}_t = r_t b_t - (\tau_k A - g_t) k_t \mathbf{1}(g_t) - \tau_k B k_t (1 - \mathbf{1}(g_t)), \\
  g_t \geq \gamma - (A - B), \\
  \text{and TVC.}
  \end{cases}
  \]
Modify model with multiple equilibria

Modified model – Transversality condition

- TVC is either

\[ b_T \leq \frac{\tau}{r_H} + \frac{(\tau_k A - g)k_T}{\rho} \equiv B_E(T), \]  

or

\[ b_T \leq \frac{\tau}{r_L} + \frac{\tau k B k_T}{\rho} \equiv B_L(T). \]  

\[ B_E(T) < B_L(T) \text{ for all } T. \]

- TVC is determined by the expectations on the path after introduction of lump-sum tax.
Modified model – Transversality condition

After imposition of lump-sum tax, debt evolves by

\[ \dot{b}_t = rb_t - \tau - \Gamma k_t, \quad \text{for} \quad t \geq T, \]

where \((r, \Gamma) = (r_H, \tau_k A - g)\) or \((r_L, \tau_k B)\), and \(k_t = k_T e^{(r-\rho)(t-T)}\).

- \((r, \Gamma) = (r_H, \tau_k A - g)\)
  
  if expectation is that technology A is dominant for \(t \geq T\).

- \((r, \Gamma) = (r_L, \tau_k B)\)
  
  if expectation is that technology B is dominant for \(t \geq T\).

Solution is

\[ b_{t+T} = x + ye^{(r-\rho)t} + (b_T - x - y)e^{rt}, \]

where \(x = \frac{\tau}{r}\),

\[ y = \frac{\Gamma k_T}{\rho}. \]
Consumer’s Transversality condition is

\[
\lim_{t \to \infty} b_{t+T} e^{-rt} = 0.
\]

TVC is equivalent to

\[
b_T \leq x + y,
\]

where \( x + y \) is either \( B_E(T) \) or \( B_L(T) \).
Modified model – Good Equilibrium

- Lump-sum tax is imposed at $T_E$, which is defined by $b_T = B_E(T)$.
- Tenure of new government is $\infty$. Credibility of fiscal policy is restored.
- Firms choose technology $A$ and new government pays $g k_t$ to firms for $t \geq T_E$, where $g = \gamma - (1 - \tau_k)(A - B)$.

\[ r_t = \begin{cases} 
  r_L = B, & \text{for } 0 \leq t < T_E, \\
  r_H = A, & \text{for } t \geq T_E, 
\end{cases} \]

\[ b_t = \begin{cases} 
  \tau k B k_0 \rho^{-1} e^{(r_L - \rho)t} + (b_0 - \tau k B k_0 \rho^{-1}) e^{r_L t}, & \text{for } 0 \leq t < T_E, \\
  \tau r_H^{-1} + (\tau k A - g) k_T \rho^{-1} e^{(r_H - \rho)t}, & \text{for } t \geq T_E, 
\end{cases} \]

\[ k_t = \begin{cases} 
  k_0 e^{(r_L - \rho)t}, & \text{for } 0 \leq t < T_E, \\
  k_T e^{(r_H - \rho)(t-T)}, & \text{for } t \geq T_E, 
\end{cases} \]

\[ c_t = \rho k_t. \]
Modified model – Bad Equilibrium

- Lump-sum tax is imposed at $T_L$, which is defined by $b_T = B_L(T)$.
- Tenure of new government becomes $\infty$.
- Too late to restore credibility of fiscal policy.
  - Proof by contradiction: Suppose fiscal policy is credible for $t \geq T_L$; firms would choose A and interest rate be $r_t = r_H$; because of high rate $r_H$, debt would increase exponentially; the equilibrium path would be unsustainable for $t \geq T_L$.
  - Firms choose technology B and new government does not pay $g_k t$ to firms for $t \geq T_L$.
Bad equilibrium:

\[ r_t = r_L = B, \quad \text{for } t \geq 0, \]

\[ b_t = \begin{cases} 
\tau_k B k_0 \rho^{-1} e^{(r_L - \rho)t} + (b_0 - \tau_k B k_0 \rho^{-1}) e^{r_L t}, & \text{for } 0 \leq t < T_L, \\
\tau r_L^{-1} + \tau k B k_T \rho^{-1} e^{(r_L - \rho)t}, & \text{for } t \geq T_L,
\end{cases} \]

\[ k_t = k_0 e^{(r_L - \rho)t}, \quad \text{for } t \geq 0, \]

\[ c_t = \rho k_t. \]
Modified model – Comparison

- **Good Equilibrium**
  - Growth rate
    \[
    \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = \begin{cases} 
    r_L - \rho, & \text{for } 0 \leq t < T_E, \\
    r_H - \rho, & \text{for } t \geq T_E. 
    \end{cases}
    \]
  - Interest rate
    \[
    r_t = \begin{cases} 
    r_L, & \text{for } 0 \leq t < T_E, \\
    r_H, & \text{for } t \geq T_E. 
    \end{cases}
    \]

- **Bad Equilibrium**
  - Growth rate
    \[
    \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_L - \rho, \quad \text{for } t \geq 0.
    \]
  - Interest rate
    \[
    r_t = r_L, \quad \text{for } t \geq 0.
    \]
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Conclusion

- We analyzed
  - effect of unsustainable public debt on technology choice and economic growth.

- We have shown
  - unsustainable debt undermines credibility of government policy because government will do whatever possible to postpone fiscal consolidation.
  - lack of commitment makes choice of technology inefficient.

- Fiscal consolidation can restore credibility and high growth (basic model).

- Fiscal consolidation may not be able to restore credibility and growth if it comes too late (second model).