Dynamics of Firms and Trade in General Equilibrium

(very preliminary)

Robert Dekle
University of Southern California

Hyeok Jeong
KDI School

Nobuhiro Kiyotaki
Princeton University

March 2012

Abstract

This paper develops a dynamic general equilibrium model that tries to reconcile the observation that aggregate movements of exports and imports are "disconnected" from real exchange rate movements, while firm-level exports co-move significantly with the real exchange rate. Firms are heterogenous, facing recurrent aggregate and firm-product specific productivity shocks, choose which goods to export, and decide to enter and exit the business endogenously. We calibrate and estimate the model with both aggregate and firm level data from Japan.
1 Introduction

Figure 1 displays the series of aggregate exports and imports together with the real exchange rate in Japan during the period of 1980-2009 in logarithmic scale. The real exchange rate is defined as the relative price between Japan’s trading partners and Japan.\(^1\) As the trading partners’ goods become relatively more expensive, we would expect that Japanese exports would increase and imports would decrease. However, such a relationship between trade and the real exchange rate is not evident in Figure 1. As Japanese real exchange rate depreciates, exports do not necessarily increase, and imports increase, which is not what we expect. During the entire sample period, the elasticity of exports with respect to the real exchange rate is -0.17, and that of imports is 0.08, although these estimates of elasticities are statistically insignificant. This lack of correlation, or correlation contrary to what we expect is an example of the so called “exchange rate disconnect puzzle,” a long standing puzzle in international macroeconomics.\(^2\)

This weak or opposite correlation between aggregate exports (or imports) and the exchange rate is observed in many other countries as well (see Hooper, Johnson, and Marquez (2000), and Dekle, Jeong, and Ryoo (2007)).\(^3\) Obstfeld and Rogoff (2000) mention that the exchange rate disconnect puzzle is one of the major puzzles in the international macroeconomics.\(^4\)

Interestingly, after the year 2000, Figure 1 shows that aggregate exports positively co-moved with the real exchange rate, but aggregate imports also positively co-moved with the real exchange rate. These co-movements during this period suggests that a general equilibrium linkage may be important in order to understand the dynamics of trade and exchange rates in

\(^1\)The real exchange rate is measured as the ratio of the weighted average of the prices of Japan’s major trading partners (in yen term) to Japanese prices, where the weights are the trading shares. The four major trading partner countries included here are the U.S., European Union, South Korea, and China and their trading shares are 0.49, 0.366, 0.095, and 0.044, respectively. (Sources: OECD Statistics)


\(^2\)This empirical puzzle was first documented by Orcutt (1950).

\(^3\)The list of other countries showing such weak correlation is Canada, France, Germany, Italy, the U.K., and the U.S.

\(^4\)Note that this “exchange rate disconnect puzzle” is different from the so called “J-curve effect.” The exchange rate disconnect puzzle is about the lack of association between the movements of exchange rates and gross export quantities while the J-curve effect is about the sluggish and J-shaped adjustment of trade balances (i.e., net export sales) in response to an improvement in the terms of trade. See Backus, Kehoe, and Kydland (1994) for the discussion of the J-curve effect.
Japan, where intermediate goods trade is dominant in imports, and increasingly more important in exports.

Recent empirical studies using firm-level data have found a more robust relationship between export and the exchange rate. In contrast to the results using aggregate data, estimates using firm level tend to find a positive relationship between depreciating exchange rates and export quantities. Among other studies, Verhoogen (2008) finds that following the 1994 peso devaluation, Mexican firms increased their exports. Fitzgerald and Haller (2008), Dekle and Ryoo (2007), and Tybout and Roberts (1997) find a positive relationship between exports and exchange rate depreciation for Irish, Japanese and Colombian firms, respectively.

Some papers have tried to reconcile these aggregate and firm level results, but mostly in a partial equilibrium framework. Dekle, Jeong, and Ryoo (2007) show that in the aggregate export equation derived by consistently aggregating the firm level export equations, where industry level productivity and firm level (instrumented) export shares are controlled for, the disconnect puzzle disappears. Berman, Martin, and Mayer (2009) use a model with heteroge-
neous firms in the spirit of Melitz (2003) to show that high productivity firms (who are heavily involved in exports) will raise their prices—that is, increase their markups—instead of increasing their export quantities in response to an exchange rate depreciation. The authors show that this selection effect of low quantity response of high productivity firms can explain the weak impact of exchange rate movements in aggregate data. There are some other recent papers that have tried to reconcile the discrepancy in a general equilibrium. Imbs and Majean (2009) and Feenstra, Russ, and Obstfeld (2010) show that the aggregation of heterogeneous industrial sectors can result in an aggregation bias in the elasticity of exports and imports with respect to exchange rates changes. Both of these papers examine only the steady-state.

In this paper, we develop a dynamic general equilibrium model with heterogeneous firms that attempts to reconcile the different responses of trade (exports and imports) to exchange rates at the aggregate- and at the firm-levels. Our model is a real business cycle model of a small open economy with a rich production structure. Firms are heterogeneous, facing recurrent aggregate and firm-product specific productivity shocks: they choose which varieties of goods to produce and export and decide to enter and exit endogenously. We calibrate and estimate our model with both aggregate and firm level data. We then carry out quantitative exercises regarding the impact of shocks to productivity and preferences on aggregate and firm-level exports and other variables of interest.\footnote{One distinguishing feature of our work is the inclusion of heterogeneous firm dynamics that is actually estimated from firm level data. In the estimation of the firm-level responses, in addition to the firm level data, we rely on the aggregate variables and moments generated from the general equilibrium model. Thus, in a sense, we provide a general equilibrium model that is integrated with a structural model of heterogeneous firm dynamics that is estimated from actual firm level data.}

We make a few choices to model heterogeneous firms to reflect our panel data of Japanese firms listed on the stock exchanges of Japan.\footnote{The raw data used here and in our paper are from almost all of the firms listed on the stock exchanges of Japan. The particular data set that we use were compiled by the Development Bank of Japan (or "Kaigin," in Japanese prior to the 2008 re-organization of government-owned enterprises, when the name of the bank was changed). Japanese listed firms cover a fairly respectable portion of the entire Japanese economy in terms of output (Fukao, et. al., 2008). In 2000, the gross sales of all the firms listed on the stock exchanges of Japan were 81 percent of Japanese nominal GDP, and 60 percent of total sales in the Japanese economy. However, listed firms are larger than the average firm in the economy. Thus, the number of listed firms account for less than 12 percent of the total number of Japanese firms, and the number of employees in listed firms are only 40 percent of all employees (Fukao, et. al., 2008).} In a well-known paper, Melitz (2003) provides a
framework where firms with different firm specific total factor productivity subject to fixed costs can generate heterogeneous exporting behavior. Das, Roberts, and Tybout (2007) provide an empirical study showing that this heterogeneity in total factor productivity among producers explains entry into and exit out of the export market, the so-called extensive margin of trade. In our Japanese panel data, there is a strong relationship between firm size and exporting status, as in Melitz (2003). The average total sales of the incumbent exporting firms is about double of the non-exporting firms. When firm level productivity is determined by a single factor of productivity, the Melitz type of trade model implies that the export share at the intensive margin (in addition to the extensive margin) be strongly correlated with firm size. Our Japanese firm level data show that this prediction is not true. The correlation between the export share and total sales is rather weak. The average correlation coefficient is only 0.08 among all firms. Among exporting firms, the correlation coefficient becomes even lower at 0.05. This weak correlation remains robust even after controlling for the industry and year effects.

Another interesting observation from Japanese firm level data is the significant presence of firms with negative profits staying in the market. About 8 percent of Japanese firms in our sample report negative profits. This fraction becomes even bigger at 11 percent among the always exporting firms, the biggest firms. Despite this presence of negative profits, Japanese listed firms do not easily exit from the business, although entry into and exit from the export market are a little more frequent.

Given these empirical observations, we choose firms to produce multiple products and are heterogeneous in terms of the productivity distribution as well as the number of the products. Firms choose which products to produce and which products to export. Thus Melitz style extensive margin adjustment is mainly at the product level (even though there are endogenous entries and exits of firms). This multi-dimensional heterogeneity helps explain the weak relationships among size, the export share and profitability in our firm-level data. Our firms also face recurrent idiosyncratic productivity shocks, and thus they may not exit with temporary
negative profits in order to enjoy the option value of continuing production.\footnote{Ghironi and Melitz (2005) analyze the dynamic effects of an aggregate productivity shock on the real exchange rate in a general equilibrium model with heterogeneous firms. But they concentrate on the extensive margin of products for export. Because there are no further idiosyncratic shocks after entry, there is no endogenous exit nor negative profits in their model.} This explains our empirical finding why Japanese firms with recurring negative profits resist to exit from their business.\footnote{More broadly, our paper is related to the recent policy literature that examines how much of a real exchange rate depreciation is necessary to close a nation’s current account imbalances. Obstfeld and Rogoff (2004) use a three-country model to calculate how much of a depreciation in the real exchange rate is needed to set the U.S. current account to zero. Dekle, Eaton, and Kortum (2008) fit their model to bilateral trade flows for 42 countries and solve for the new equilibrium in real exchange rates to eliminate all current account imbalances.}

In Section 2, we present a model of small open economy and study the equilibrium dynamics and steady state of the model economy. In Section 3, we calibrate the model. In Section 4, aggregate dynamics is simulated. Section 5 concludes.

## 2 Small Open Economy Model

### 2.1 Set-up

There is a continuum of home firms $h \in \mathcal{H}_t$ each of which produces $I_{ht}$ number of differentiated products for the home and export markets at date $t$. Firm $h$ produces a differentiated product $q_{hit}^H$ for the home market from labor $l_{hit}^H$ and imported input $m_{hit}^H$, according to a constant returns to scale technology

$$q_{hit}^H = a_{hit} Z_t \left( \frac{l_{hit}^H}{\gamma_L} \right)^{\gamma_L} \left( \frac{m_{hit}^H}{1 - \gamma_L} \right)^{1-\gamma_L}, \text{ for } i = 1, 2, \ldots, I_{ht},$$

where $a_{hit}$ is the productivity of firm $h$ to produce the differentiated product $(h, i)$ at date $t$ and $Z_t$ is the aggregate productivity shock and $\gamma_L \in (0, 1)$ is the labor share. Because no two firms produce the same product, each differentiated product is indexed by $(h, i)$. Producing a
differentiated product for the export market has the same marginal productivity as producing for the home market, but requires a fixed cost for each variety,

\[ q_{hit}^F = a_{hit}Z_t \left[ \left( \frac{l_{hit}^F}{\gamma_L} \right)^{\gamma_L} \left( \frac{m_{hit}^F}{1 - \gamma_L} \right)^{1-\gamma_L} - \phi \right], \text{ for } i = 1, 2, \ldots I_{ht}. \]

We assume that the fixed cost for exporting a differentiated product is constant in terms of input composite - Cobb-Douglas function of labor and imported input.

Home final goods are produced from a variety of differentiated products according to a constant returns to scale CES production function

\[ Q_t^H = \left[ \int_{h \in H_t} \left( \sum_{i=1}^{I_{ht}} q_{hit}^H \phi \right) dh \right]^{-\frac{\theta}{\nu-1}}, \]

where \( \theta > 1 \) is the elasticity of substitution between among goods. Home output for export \( Q_t^F \) is produced from home differentiated goods

\[ Q_t^F = \left[ \int_{h \in H_t} \left( \sum_{i=1}^{I_{ht}} q_{hit}^F \phi \right) dh \right]^{-\frac{\theta}{\nu-1}}. \]

Any new entrant who pays a sunk cost \( \kappa_E \) at date \( t \) draws an opportunity of producing \( b \in \{1, 2\} \) number of new products from date \( t+1 \), where

\[ b = \begin{cases} 2, \text{ with probability (wp.) } \iota, \\ 1, \text{ wp. } 1 - \iota. \end{cases} \]

Thus the average number of new products drawn is equal to \( 1 + \iota \). The productivity of each new product is independently and identically distributed: \( a_{hit} = 0 \) with probability \( 1 - \lambda' \) and \( a_{hit} \) is distributed according to a Pareto distribution with parameter \((1, \alpha)\) with probability \( \lambda' \). That is

\[ a_{hit} = \begin{cases} \in [1, a], \text{ wp. } \lambda F(a) = \lambda' (1 - a^{-\alpha}), \\ 0, \text{ wp. } 1 - \lambda'. \end{cases} \]

The probability density function of the Pareto distribution is

\[ f(a) = a^\alpha (\alpha + 1), \text{ for } a \in [1, \infty). \]

Any one can enter by paying the sunk-cost, and new entrants are heterogeneous in terms of number of differentiated products (width \( b \)) and the distribution of productivity of products (height \( a_{hit+1} \)).
We assume

\[ \lambda \equiv (1 + \iota)\lambda' < 1, \quad \text{(Assumption 1)} \]
\[ \alpha > 1 \text{ and } \alpha > \theta - 1. \quad \text{(Assumption 2)} \]

Assumption 1 implies that the average number of products with positive productivity is less than unity per new draw, and that some new entrants fail to obtain the productive new products. Assumption 2 says that the fraction of products with high productivity is relatively small, which later guarantees that CES production function of final goods is well behaved.

The firm who has existing products must pay the fixed maintenance cost \( \kappa \) (in terms of home final goods) for each product in order to produce and maintain its productivity. (The firm who wants to maintain \( I_{ht} \) number of products must pay \( \kappa \cdot I_{ht} \)). The product which the firm pays the fixed cost has the same productivity in the next period (\( a_{hit+1} = a_{hit} \)) with probability \( 1 - \delta \), and receives a new productivity draw according to the same distribution as a new entrant with probability \( \delta \). Thus the number of products each firm produces may increase or decrease depending upon the new draw of \( b \), and the distribution of productivity of producible differentiated products changes depending upon the new draw of \( a_{hit+1} \). Because firms are heterogeneous in terms of the number and the productivity of their products, we can show that there are only weak relationships among size, the export share and profitability across firms - a feature of our Japanese firm-level data. If the firm does not pay the fixed cost \( \kappa \) for an existing product, it loses the technology for this product for sure and forever.

Home final goods are either consumed by households and government, or used for new entry and maintenance of the existing technology as

\[ Q^H_t = C_t + G_t + \kappa E N_{Et} + \kappa N_t, \]

where \( C_t \) and \( G_t \) are consumption of households and government, \( N_{Et} \) is the measure of entering firms and \( N_t \) is the measure of existing differentiated products which firms try to maintain. We can think the cost of drawing new technology and maintaining old technology as investment in intangible capital. On the other hand, we abstract from tangible capital and tangible capital investment.
The representative household supplies labor $L_t$ to earn wage income, consumes final goods $C_t$ and holds home and foreign real bonds $D^{H}_t$ and $D^{*H}_t$ to maximize its expected utility,

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t - \psi_0 \frac{L^{1+1/\psi}_t}{1+1/\psi} + \xi^{*H}_t \ln D^{*H}_t \right),$$

subject to the budget constraint

$$C_t + \kappa E_N E_t + \kappa N_t + D^{H}_t + \epsilon_t D^{*H}_t = w_{Lt} L_t + \Pi_t + R_{t-1} D^{H}_{t-1} + \epsilon_t R_{t-1} D^{*H}_{t-1} - T_t. \quad (1)$$

(We index goods by the origin and user countries. For the origin, we label goods by naught for home and by * for foreign. For the user, we label goods by $H$ for home and by $F$ for foreign. For example, $D^{*H}_t$ is foreign bond that are held in the home country.) $w_{Lt}$ is real wage rate, $\epsilon_t$ is the real exchange rate (the relative price of foreign and home final goods), and $R_t$ and $R^*_t$ are home and foreign real gross interest rates. $T_t$ is lump-sum tax and $\Pi_t$ is the real gross profits distribution from businesses, while the net profit is $\Pi_t - \kappa N_t$. Home and foreign bonds are used as means of saving. In addition we assume that the holding of foreign bonds facilitates transactions and provides utility. The utility from holding foreign bonds is subject to the "liquidity" shock, $\xi^{*H}_t$.\(^{10}\)

We assume that all home imports are inputs to production, and that the imported input price is normalized to be one in terms of foreign final goods. We assume that foreign aggregate demand for home exports are given by

$$Q^F_t = (p^F_t)^{-\phi} Y^*_t,$$

where $Y^*_t$ is an exogenous foreign demand parameter, $p^F_t$ is an endogenous export price in terms of foreign final goods, and $\phi > 0$ is the constant elasticity of demand for home exports. We assume that foreigners do not hold home bond. Then foreign bond holdings $D^{*H}_t$ of the home representative household evolves along with exports and imports as

$$D^{*H}_t = R^*_{t-1} D^{*H}_{t-1} + p^F_t Q^F_t - M^{*H}_t. \quad (2)$$

\(^{10}\)The idea is similar to money in utility function. Section 5.3.8 of Obstfeld and Rogoff (1998) presents a model with both home and foreign money in the utility function to analyze the phenomenon of dollarization. We ignore the utility of home bonds for simplicity.
where \( M^*_H = \int_{h \in \mathcal{H}_t} \left[ \sum_{i=1}^{l_{ht}} (m^*_H + m^*_F) \right] dh \) are total imported input of the home country. The government budget constraint is given by

\[
D^*_H = R_{t-1} D^*_H + G_t - T_t. \tag{3}
\]

Here, because the foreigners do not hold home bond, the home bond holding of the representative household is equal to the government bond supply. Government adjusts tax to stabilize the outstanding debt

\[
T_t - T = \zeta_T \left[ R_{t-1} D^*_H - RD^*_H \right], \tag{4}
\]

for an autonomous choice of the expenditure \( G_t \), where \( T \) and \( RD^*_H \) are tax and government debt at the beginning of period in the steady state.

### 2.2 Competitive Equilibrium

All markets for the factors of production and outputs are perfectly competitive, except that the market for differentiated products are monopolistically competitive.

Consistent with the usual CES production function of final goods manufactured from differentiated products, each firm faces a downward sloping demand curve for its product in home and foreign markets as a function of prices \( p^*_H \) and \( p^*_F \), such that

\[
q^*_H = \left( \frac{p^*_H}{p^*_t} \right)^{-\theta} Q^*_H, \\
q^*_F = \left( \frac{p^*_F}{p^*_t} \right)^{-\theta} Q^*_F,
\]

where \( p^*_H \) and \( p^*_F \) are the price indices of home final output for the home and export markets

\[
1 = p^*_H = \left[ \int_{h \in \mathcal{H}_t} \left( \sum_{j=1}^{l_{ht}} \left( p^*_H \right)^{1-\theta} \right) dh \right] \frac{1}{1-\theta}, \tag{5}
\]

\[
p^*_F = \left[ \int_{h \in \mathcal{H}_t} \left( \sum_{j=1}^{l_{ht}} \left( p^*_F \right)^{1-\theta} \right) dh \right] \frac{1}{1-\theta}.
\]

We use home final goods as the numeraire in the home market, and foreign final goods as the numeraire in the foreign market.
Note that the production functions of differentiated products all have a common component: the Cobb-Douglas function of input composite. Moreover the ratio of labor to imported inputs is equal across firms when firms minimize the costs under perfectly competitive factor market.

Let $x_{hit}^H$ and $x_{hit}^F$ be input composite used for producing a given differentiated product for the home and export markets. Then the production functions can be simplified as

$$q_{hit}^H = a_{hit} Z_t \cdot x_{hit}^H,$$

$$q_{hit}^F = a_{hit} Z_t \cdot (x_{hit}^F - \phi).$$

Aggregate production of the input composite is equal to the sum of input composite use

$$X_t = \left( \frac{L_t}{\gamma_L} \right)^{\gamma_L} \left( \frac{M_t^{*H}}{1 - \gamma_L} \right)^{1 - \gamma_L} = \int_{h \in H_t} \left( \sum_{i=1}^{l_{hit}} (x_{hit}^H + x_{hit}^F) \right) dh.$$

Because the price of imported inputs at home is equal to the real exchange rate (due to our choice of numeraire), cost minimization implies that the unit cost of the input composite $w_t$, and the demand for imported inputs satisfy

$$w_t = (w_{Lt})^{\gamma_L} \epsilon_t\gamma_L,$$

$$M_t^{*H} = (1 - \gamma_L) \frac{w_t X_t}{\epsilon_t}.$$  \(6\)

After maximizing current profits, each firm sets price as a mark-up over their unit production cost

$$P_{hit}^H = \frac{\theta}{\theta - 1} \frac{w_t}{a_{hit}Z_t} \equiv p_t^H (a_{hit}),$$

$$P_{hit}^F = \frac{\theta}{\theta - 1} \frac{w_t/\epsilon_t}{a_{hit}Z_t} \equiv p_t^F (a_{hit}),$$

for all products produced. The quantity of each product produced for home and foreign market depends only upon the productivity (aside from the aggregate variables).

$$q_{hit}^H = \left( \frac{p_t^H (a_{hit})}{p_t^H} \right)^{-\theta} Q_t^H \equiv q_t^H (a_{hit}),$$

$$q_{hit}^F = \left( \frac{p_t^F (a_{hit})}{p_t^F} \right)^{-\theta} Q_t^F \equiv q_t^F (a_{hit}),$$

11
Although each firm may produce multiple differentiated products, the firm can decide how to produce and whether to continue producing each product independently from their choice of the other products.\footnote{The founder of Kyocera, Mr Kazuo Inamori, proposes an "amoeba" management style, in which each production unit makes relatively independent production decisions, while the number of production units multiply and shrink like "amoebas." Our technology can be seen as a justification for the "amoeba" management style. See also Bernard, Redding and Schott. (2010, 2011).}

We conjecture that in equilibrium, all firms choose to pay the fixed maintenance cost with positive productivity. Then the total measure of products with positive productivity evolves with the maintenance and new entries as

\[ N_{t+1} = (1 - \delta + \delta \lambda) N_t + \lambda N_{E_t}. \]  

(8)

Because of Assumption 1, \( N_t \) does not grow without limit. Let \( N_t(a) \) be the measure of products with productivity \( a \). Then from the specific feature of our idiosyncratic productivity evolution, \( N_t(a) \) is a proportional to \( N_t \) as:

\[ N_t(a) = f(a)N_t. \]

Then from (5) and (7), the price index for home final goods for the home market becomes

\[ \frac{1}{1} = p_t^H = \left[ \int_1^\infty p_t^H(a) \right]^{\frac{1}{1-\theta}} N_t f(a) da = \frac{\theta}{\theta - 1} \frac{w_t}{A_t^H}, \]

where

\[ A_t^H \equiv \bar{a} N_t \frac{1}{\theta-1} Z_t \]

and

\[ \bar{a} \equiv \left( \int_1^\infty a^{\theta-1} f(a) da \right)^{\frac{1}{\theta-1}} = \left( \frac{\alpha}{\alpha + 1 - \theta} \right)^{\frac{1}{\theta-1}} \]

\( A_t^H \) is the aggregate productivity of home firms for home market, and \( \bar{a} \) is the average productivity of products that are produced. Thus the unit cost of input composite is given by

\[ w_t = \frac{\theta - 1}{\theta} A_t^H = w(N_t, Z_t). \]

(9)

We conjecture that there is a lowest productivity level \( a_t > 1 \) at which the product makes zero profit for export:

\[ \pi_t^F(a_t) = \epsilon_t q_t^F(a_t) q_t^F(a_t) - w_t \left[ \frac{q_t^F(a_t)}{a_t Z_t} + \phi \right] = w_t \left[ \frac{1}{\theta - 1} \frac{q_t^F(a_t)}{a_t Z_t} - \phi \right] = 0, \]

or

\[ q_t^F(a_t) = (\theta - 1) \phi a_t Z_t. \]
Thus only a fraction
\[ n_t^F = \text{Prob}(a \geq \underline{a}_t) = (\underline{a}_t)^{-\alpha} < 1 \]  
(10)
of products are exported.

Price index of home final output for foreign market is
\[ p_t^F = \frac{\theta}{\theta - 1} \frac{w_t/\epsilon_t}{\overline{\sigma}_t^{\theta} N_t^{\frac{1}{\theta-1}} Z_t} = \frac{1}{\epsilon_t \overline{\sigma}_t^F}, \]
where
\[ \overline{\sigma}_t^F = \left[ \int_{\underline{a}_t}^{\infty} a^{\theta-1} f(a) da \right]^{\frac{1}{\theta-1}} = \left[ \frac{\alpha}{\alpha + 1 - \theta} (\underline{a}_t)^{\theta-\alpha-1} \right]^{\frac{1}{\theta-1}} = \overline{a}_t (\underline{a}_t)^{-\frac{\alpha + 1 - \theta}{\theta - 1}}. \]
This home firms’ productivity measure for export \( \overline{\sigma}_t^F \) is a decreasing function of the lower bond of productivity for export \( \underline{a}_t \).

The export quantity index becomes
\[ Q_t^F = \left[ \int_{\underline{a}_t}^{\infty} q_t^F(a)^{\frac{\theta-1}{\theta}} f(a) N_t da \right]^{\frac{1}{\theta-1}} = q_t^F(\underline{a}_t) N_t^{\frac{\theta}{\theta-1}} \left[ \int_{\underline{a}_t}^{\infty} \left( \frac{a}{\underline{a}_t} \right)^{\theta-1} f(a) da \right]^{\frac{1}{\theta-1}} = (\theta - 1) \phi Z_t N_t^{\frac{\theta}{\theta-1}} \alpha^\theta (\underline{a}_t) \frac{\alpha^{\theta+1-\theta}}{\theta-1}. \]

To derive the second equation, we use the property \( q_t^F(a)/q_t^F(\underline{a}_t) = (p_t^F(a)/p_t^F(\underline{a}_t))^{-\theta} = (a/\underline{a}_t)^\theta. \) From the export demand condition, we also have
\[ Q_t^F = (p_t^F)^{-\phi} Y_t^* = (\underline{a}_t)^{-\frac{\alpha + 1 - \theta}{\theta - 1}} e^{\phi} Y_t^*. \]
Thus the export market clearing condition implies
\[ \underline{a}_t = \frac{\alpha (\theta - 1) \phi A_t^H N_t^{\frac{\alpha - 1}{(\theta - 1) (\theta - 1)}}}{\alpha + 1 - \theta} \frac{\phi}{e^{\phi} Y_t^*} \approx a \left( \frac{\phi Y_t^*}{A_t^H N_t} \right), \text{ where } a' (\cdot) < 0. \]  
(11)
For \( a_t > 1 \) so that the extensive margin for export to exist, we need
\[ \frac{\phi Y_t^*}{A_t^H N_t} < \frac{\alpha (\theta - 1) \phi}{\alpha + 1 - \theta}. \]
(Condition 1)
If this condition were violated, then all home firms would export all the products, but this is not consistent with the observation. Thus we restricts the attention to the case in which Condition 1 is satisfied in the following.

The export value in terms of home final goods is

\[ \epsilon_t p_t^F Q_t^F = \left( \epsilon_t p_t^F \right)^{1-\varphi} \epsilon^\varphi Y_t^* = \frac{A_t}{\varphi} \frac{\left( \alpha + 1 - \theta \right) \phi}{\alpha + 1 - \theta} N_t \left[ \frac{(\alpha+1-\theta)(1-\varphi)}{\alpha(\theta-\varphi)-(\theta-1)(1-\varphi)} \right] (\epsilon_t^\varphi Y_t^*)^{\alpha(\theta-1)} \]  

(12)

### 2.3 Disconnect Between the Aggregate and Firm Level Responses

Because the number of products produced \( N_t \) is a state variable, the extensive margin \( Q_t \) reacts to an exogenous shift in foreign demand and the real exchange rate. When the elasticity of foreign demand for home products is relatively small compared to the elasticity of substitution among differentiated products (\( \varphi \) is small relative to \( \theta \)), then the export quantity \( Q_t^F \) and export value (in terms of home final goods) are relatively insensitive to the real exchange rate shift. In contrast, because both intensive and extensive margins adjust in the short run, the exports of the products whose productivities are close to the lower bound for export is sensitive to the real exchange rate shift. As in Green (2009), the exports of the low productivity products drop like "flies" when there is an adverse shock such as a real exchange rate appreciation.

Our Japanese firm-level data (Kaigin data) are mostly of relatively large firms, which typically produce multiple products - possibly after a number of good new draws of \( b = 2 \). If a majority of products of some firm is close to the lower bound for export, then the export of this firm is sensitive to the real exchange rate shifts (which moves the lower bound). Because such firms are common under Assumption 2, the firm-level export tends to react significantly to the real exchange rate. In contrast, the products with considerably higher productivity than the lower bound is not very sensitive to the real exchange rate shifts, and their share in the aggregate export is large. Thus the aggregate exports are less sensitive contemporaneously to the real exchange rate shift. This heterogeneous reaction of exports to the real exchange rate
shift across different products explains why firm level exports co-move significantly with the real exchange rate, while aggregate exports appear "disconnected" from the real exchange rate.\footnote{Our explanation of the extensive margin adjustment at product level is consistent with Dekle, Jeong and Ryoo (2007), which find that the apparent lack of relationship between the exchange rate and aggregate exports occur through the intensive margin of export sales within firms, rather than through the extensive margin of entry and exit of firms in the export market.}

### 2.4 Equilibrium Dynamics

The aggregate composite input use for export is

\[
X_t^F = \int_{\alpha_t}^{\infty} \left[ \frac{q_t^F(a)}{aZ_t} + \phi \right] f(a)N_t da
= \int_{\alpha_t}^{\infty} \phi \left[ \left( \frac{a}{a_t} \right)^{\theta - 1} (\theta - 1) + 1 \right] f(a)N_t da
= \phi \frac{\theta \alpha + 1 - \theta}{\alpha + 1 - \theta} (a_t)^{-\alpha} N_t. \tag{13}
\]

Aggregate composite input use for home final goods is proportional to the home final goods output as

\[
X_t^H = \int_{1}^{\infty} \frac{q_t^H(a)}{aZ_t} f(a)N_t da
= \frac{Q_t^H}{A_t^H}.
\]

The labor supply condition of the household is given by

\[
w_{Lt} = \psi_0 L_t^{\frac{1}{\gamma L}} C_t.
\]

Together with (6), we have

\[
L_t = \frac{1}{(\psi_0 C_t)^\psi} \left( \frac{w_t}{\epsilon_t} \right)^{\frac{\psi}{\gamma L}},
\]

\[
X_t = \left( \frac{w_t}{\epsilon_t} \right)^{\frac{1 - \gamma L}{\gamma L}} L_t^{\frac{1}{\gamma L}}
= \frac{1}{\gamma L (\psi_0 C_t)^\psi} \left[ \frac{w_t^{1 - \gamma L + \psi}}{\epsilon_t^{(1 - \gamma L)(1 + \psi)}} \right]^{\frac{1}{\gamma L}}
= X_t^H + X_t^F. \tag{14}
\]
The profit arising from selling a product with productivity \( a_{hit} = a \) in the home market is

\[
\pi_t^H(a) = p_t^H(a)q_t^H(a) - w_t x_t^H(a)
\]

\[
= \frac{1}{\theta - 1} w_t x_t^H(a),
\]

i.e., the net mark-up times the input costs. The profits arising from exporting a product with productivity \( a_{hit} = a \geq a_0 \) is

\[
\pi_t^F(a) = \epsilon_t p_t^F(a)q_t^F(a) - w_t x_t^F(a)
\]

\[
= w_t \left[ \frac{1}{\theta - 1} x_t^F(a) - \phi \right].
\]

Let \( V_t(a) \) be the values of the products with productivity \( a \) at the beginning of this period (who decides to pay the fixed cost of maintenance). The Bellman equation is

\[
V_t(a) = \pi_t^H(a) + \max\{\pi_t^F(a), 0\} - \kappa
\]

\[
+ E_t \Lambda_{t,t+1} \left[ (1 - \delta)V_{t+1}(a) + \delta \lambda \int_1^\infty V_{t+1}(a') f(a') da' \right],
\]

where \( \Lambda_{t,t+1} = \beta C_t/C_{t+1} \). The average value of the products produced is given as

\[
\overline{V}_t \equiv \int_1^\infty V_t(a) f(a) da
\]

\[
= \pi_t - \kappa + (1 - \delta + \delta \lambda) E_t (\Lambda_{t,t+1} \overline{V}_{t+1}),
\]

where \( \pi_t \) is the average profit of the products with positive productivity:

\[
\pi_t = \int_1^\infty \left\{ \pi_t^H(a) + \max\{\pi_t^F(a), 0\} \right\} f(a) da
\]

\[
= w_t \left[ \frac{X_t}{(\theta - 1) N_t} - \phi n_t^F \right].
\]

The free entry condition for a potential entrant is

\[
\kappa_E = \lambda E_t \left( \Lambda_{t,t+1} \overline{V}_{t+1} \right).
\]

The necessary and sufficient condition that the firm strictly prefers to maintain a product with the lowest productivity by paying the fixed cost is

\[
0 < V_t(1) = \pi_t^H(1) - \kappa + E_t \left[ (1 - \delta)V_{t+1}(1) + \delta \lambda \overline{V}_{t+1} \right], \text{ for all } t
\]
or

\[ 0 < \pi^H_t(1) - \kappa + \delta \lambda \kappa E. \quad \text{(Condition 2)} \]

Notice that this condition is satisfied even if realized current net profits of each product is negative \((\pi^H_t(1) < \kappa)\), because there is an option value for the low productivity product to become a high productivity product.\(^{13}\) This helps explain why firms often record negative profits after paying their fixed costs of maintaining the business. In addition, because firms may have a large number of low productivity products, there is only a weak correlation between size and profitability across firms - another curious aspect of Japanese firms.

The final goods market clearing implies

\[ C_t + G_t + \kappa E N_{Et} + \kappa N_t = A^H_t X^H_t. \quad \text{(19)} \]

From (2), (6) and (12), net foreign assets evolve as

\[ D^*_t = R^*_t - (1 - \gamma_L) \frac{w_t X_t}{\epsilon_t} \]

\[ + \left[ \frac{\alpha (\theta - 1) \phi}{\alpha + 1 - \theta} A^H_t N_t \right]^{(\alpha + 1 - \theta)(1 - \varphi)} \frac{Y_t^{* \alpha (\theta - 1)}}{\epsilon_t^{(\alpha + 1 - \theta)(1 - \varphi)}}. \quad \text{(20)} \]

From the utility maximization of the household, we have

\[ 1 = R_t E_t (A_{t,t+1}) \quad \text{(21)} \]

and

\[ \epsilon_t - R^*_t E_t (A_{t,t+1} \epsilon_{t+1}) = \xi^*_t \frac{C_t}{D^*_t}. \quad \text{(22)} \]

The first equation is a standard Euler equation for home bond. The left hand side (LHS) of the second equation is the opportunity cost of holding one unit of the foreign bond. The right hand side (RHS) is the marginal rate of substitution between foreign bond holdings and consumption.

\(^{13}\)The option value due to the idiosyncratic productivity shock cannot be too large, because we conjecture that the firm will not maintain the product with zero productivity. The condition for firms not to maintain zero productivity product is

\[ 0 > -\kappa + \delta \lambda \kappa E. \]
The aggregate state of our small open economy is described by the state variables $\mathcal{M}_t = (N_t, D_{t-1}^H, D_{t-1}^H, Z_t, G_t, \xi_t^*, Y_t^*, R_t^*)$, where the first three are endogenous and the last five are exogenous. The market equilibrium condition (3), (4), (8) - (11) and (13) - (22) determine $(w_t, \alpha_t, n_t^F, X_t, X_t^H, X_t^F, C_t, \epsilon_t, R_t, \bar{V}_t, \pi_t, N_{Et}, N_{t+1}, D_t^*, D_t^H, T_t)$ as a function of the state variables. The budget constraint (1) is automatically satisfied once all the market clearing conditions are satisfied (by a variant of Walras’ Law), noting that aggregate profit is equal to the average profit multiplied by the number of products produced ($\Pi_t = \pi_t N_t$).

2.5 Dynamics of the "Shrunk" Model

We first examine the market clearing condition for net foreign assets (22) to fix intuition. Suppose as it is likely, that a liquidity shock to foreign bonds $\xi_t^*$ is very volatile in the short-run. The supply of net foreign assets changes sluggishly over time through the current account. Consumption is relatively smooth by permanent income theory if labor supply is relatively inelastic and the investment on technology $(\kappa_{E} N_{Et} + \kappa N_t)$ serves as a buffer to absorb shocks (which we have to verify later). Then since the liquidity shock to foreign bonds appears only in the market clearing condition for net foreign assets, the real exchange rate has to adjust quickly to the volatile liquidity shock at high frequency - even though at low frequency, the adjustment of the current account and consumption are as important as the real exchange rate adjustment.

Thus in our economy, the high frequency movement of the real exchange rate is dominated by the liquidity shock. Empirically, we can treat the short-run movement of the real exchange rate as almost "exogenous", because we can always find a liquidity shock to justify the observed movement of real exchange rates as long as our boundary conditions (Condition 1) and (Condition 2) are satisfied and the evolution of net foreign assets is stable in the long-run. In addition, net foreign assets appear only in the equation for the evolution of net foreign assets (20).

Therefore, below we consider the following abbreviated or "shrunk" model. We take $\mathcal{M}'_t = (N_t, D_{t-1}^H, Z_t, G_t, Y_t^*, \epsilon_t^*)$ as the state variables, and solve for the fourteen endogenous variables $(w_t, \alpha_t, n_t^F, X_t, X_t^H, X_t^F, C_t, R_t, T_t, \bar{V}_t, \pi_t, N_{Et}, N_{t+1}, D_t^H)$ as functions of the state vari-
ables which satisfy the fourteen equilibrium conditions (3), (4), (8) - (11), (13) - (19) and (21).

Here there are only two endogenous state variables, \( N_t \) and \( D_{t-1}^H \), which greatly simplifies our analysis.

Equation (12) provides the expression for the value of total exports as explicit functions of the state variables. Our discussion of the adjustment of exports through the extensive and intensive margins can then be made clearer, because we can now take the real exchange rate as exogenous. Moreover from (16) and (18), we have

\[
\nabla_t = \pi_t - \kappa + (1 - \delta + \delta \lambda) \frac{K_E}{\lambda}.
\]

Multiplying both sides of this expression at date \( t+1 \) by \( \Lambda_{t,t+1} \), we have

\[
\frac{K_E}{\lambda} = E_t \Lambda_{t,t+1} \nabla_{t+1} = E_t \left\{ \Lambda_{t,t+1} \left[ \pi_{t+1} - \kappa + (1 - \delta + \delta \lambda) \frac{K_E}{\lambda} \right] \right\},
\]

or

\[
\kappa_E [1 - (1 - \delta + \delta \lambda) E_t (\Lambda_{t,t+1})] = \lambda E_t [\Lambda_{t,t+1} (\pi_{t+1} - \kappa)].
\]

The LHS is the cost of entering now instead of the next period, and the RHS is the expected net profits of the next period which the firm would lose by delaying entry. (Remember the entering firm can only produce from the next period). Using (17), this can be written as

\[
\kappa_E \left[ 1 - (1 - \delta + \delta \lambda) E_t \left( \beta \frac{C_t}{C_{t+1}} \right) \right] = \lambda E_t \left( \beta \frac{C_t}{C_{t+1}} \left\{ -\kappa + \frac{\theta - 1}{\theta} A_t^H \left[ \frac{X_{t+1}}{(\theta - 1)N_{t+1}} - \phi \left[ a \left( \frac{\epsilon_{t+1} Y_{t+1}}{A_{t+1}^H N_{t+1}} \right) \right]^{-\alpha} \right] \right\} \right),
\]

where \( A_t^H \equiv \bar{a} N_t \bar{r}_t Z_t \). Supply condition for aggregate composite input is

\[
X_t = \frac{1}{\gamma L (\psi_0 C_t)^\psi} \left[ \left( \frac{\theta - 1}{\theta} A_t^H \right)^{1-\gamma L + \psi} \right]^{\frac{1}{\gamma L}}.
\]

The goods market clearing condition can be written as

\[
C_t + \kappa N_t + \frac{K_E}{\lambda} [N_{t+1} - (1 - \delta + \delta \lambda) N_t] + G_t
\]

= \[
A_t^H \left\{ X_t - \frac{\alpha \theta + 1 - \theta}{\alpha + 1 - \theta} \phi \left[ a \left( \frac{\epsilon_t^\varphi Y_t^*}{A_t^H N_t} \right) \right]^{-\alpha} N_t \right\}
\]

\[23\]
\[(C_t, X_t, N_{t+1})\] solves (23, 24, 25) as a function of \((N_t, Z_t, G_t, Y^*_t, \epsilon_t)\). Then we can find \((R_t, D^H_t, T_t)\) which satisfies (21, 3, 4). We have to verify that the conditions (Condition 1) and (Condition 2) are satisfied in equilibrium. (Condition 2) can be written as
\[
\kappa - \delta \kappa_E < \frac{\alpha + 1 - \theta}{\alpha \theta} A_t^H \left\{ \frac{X_t}{N_t} - \frac{\alpha \theta + 1 - \theta}{\alpha + 1 - \theta} \phi \left[ \alpha \left( \frac{e_t^\phi Y^*_t}{A_t^H N_t} \right)^{-\alpha} \right] \right\}.
\]

Using (6), (13) and (15), home real GDP is given by
\[
Y_t = Q_t^H + \epsilon_t p_t^F Q_t^F - \epsilon_t M_t^*H
= w_t \left\{ \frac{\theta}{\theta - 1} X_t - \phi \left( \frac{\phi^\phi_t Y_t^{*+1}}{A_t^H N_{t+1}} \right)^{-\alpha} \right\} N_t - (1 - \gamma_L)X_t
= \bar{a} N_{t+1}^{\frac{1}{1-\phi}} Z_t \left[ \gamma_L \frac{\theta - 1}{\theta} X_t + \frac{1}{\theta} X_t - \phi \left[ \frac{\phi^\phi_t Y_t^{*+1}}{A_t^H N_{t+1}} \right]^{-\alpha} N_t \right].
\]
The first term of RHS is wage, the second is gross profit and the last is the fixed cost for export.

### 2.6 Steady State of the Full Model

From the free entry condition with \(\Lambda_{t,t+1} = \beta\) in the steady state, we have
\[
\kappa_E [1 - (1 - \delta + \delta \lambda) \beta] = \lambda \beta (\bar{\pi} - \kappa),
\]
or
\[
\bar{\pi} = \kappa + \frac{\kappa_E}{\lambda} \left( \frac{1}{\beta} - 1 + \delta - \delta \lambda \right)
= \frac{w X}{(\theta - 1) N} - w \phi a^{-\alpha}
\]
(26)

From (22), we have
\[
\epsilon D^H = \frac{1}{1 - \beta R^*} \xi^*H C.
\]
Together with (20), we have
\[
(R^* - 1) \epsilon D^H = \frac{R^* - 1}{1 - \beta R^*} \xi^*H C
= (1 - \gamma_L) w X - a \frac{\alpha + 1 - \alpha}{\alpha + 1 - \alpha (1 - \varphi)} e^\phi Y^*.
\]
Then

\[
(1 - \gamma_L) \frac{wX}{N} = \frac{a^{\frac{1-\theta}{\theta}}(1-\varphi) \xi^\varphi Y^*}{N} + \frac{R^* - 1}{1 - \beta R^*} \xi^* H C \frac{1}{N}.
\]  

(27)

From (11), we get

\[
\frac{a^{\frac{\alpha(\theta-\varphi)-(\theta-1)(1-\varphi)}}{\theta-1}}{wN} = \frac{\alpha \phi \theta}{\alpha + 1 - \theta} \frac{wN}{\xi^\varphi Y^*}.
\]  

(28)

From (26) – (28), we have

\[
\frac{\alpha \phi \theta}{\alpha + 1 - \theta} w = a^\alpha a^{\frac{1-\theta}{\theta-1} (1-\varphi)} \xi^\varphi Y^* \frac{wN}{N} = a^\alpha \left\{ (1 - \gamma_L)(\theta - 1) \left( \frac{\pi}{\theta} + w \phi a^{-\alpha} \right) - \frac{R^* - 1}{1 - \beta R^*} \xi^* H C \frac{1}{N} \right\}
\]

or

\[
\left[ \frac{\alpha}{\alpha + 1 - \theta} - (1 - \gamma_L) \frac{\theta - 1}{\theta} \right] \frac{wX}{N} + \frac{R^* - 1}{1 - \beta R^*} \xi^* H \theta - 1 \frac{C}{N} = \alpha(\theta - 1) \frac{1}{\alpha + 1 - \theta} \frac{1}{\pi}.
\]  

(29)

From (25), we have

\[
\frac{C}{N} + \kappa + \frac{\kappa E}{\lambda} \delta (1 - \lambda) + \frac{G}{N} = A \frac{H X}{N} - A \frac{H \phi a^{-\alpha} \theta + 1 - \theta}{\alpha + 1 - \theta},
\]

or using (26)

\[
\left[ \frac{\alpha \theta + 1 - \theta}{(\theta - 1)(\alpha + 1 - \theta)} - 1 \right] \frac{wX}{N} + \frac{\theta - 1}{\theta} \frac{C}{N} = \left( \frac{\alpha \theta + 1 - \theta}{\alpha + 1 - \theta} - \frac{\theta - 1}{\theta} \right) \frac{\pi}{\theta} + \frac{\theta - 1}{\theta} \left[ \frac{\kappa_E}{\lambda} \left( \frac{1}{\beta} - 1 \right) - g \right],
\]  

(30)

where \( g = G/N \) is government purchase per measure of products.

We can solve (29) and (30) simultaneously with respect to \( \frac{C}{N} \) and \( \frac{wX}{N} \) as a function of parameters. Also from (24), we know

\[
\gamma_L(\psi_0 \frac{C}{N})^\psi \frac{wX}{N} = \left[ \frac{\varphi_0 - 1}{\beta} N \frac{1}{\psi} \right]^{1+\psi}.
\]  

(31)

Thus we can find the steady state value of \( N \). Then all the endogenous variables in steady state are determined as functions of exogenous parameters.

3 Calibration
4 Simulation

5 Conclusion

References


