A Financial Crisis in a Monetary Economy

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Introduction

Two issues in the research of financial crises

- Long-term decline in the aggregate productivity after a financial crisis
  - 13 percent out of 18 percent TFP decline in the Great Depression unexplained (Ohanian 2001)
  - The lost decade of Japan (Hayashi and Prescott 2002)
  - Debt Disorganization (小林・加藤 2001, Kobayashi and Inaba 2004)

- Can monetary policy resolve the damage of a financial crisis?
  - Theoretically yes (Allen and Gale 1998, Diamond and Rajan 2006),
  - Is the Friedman rule optimal?
  - Balance sheet problem could not be resolved by monetary easing in Japan.
Hypothesis:

- The generalization of the Lagos-Wright monetary framework.
- Two production technology: High and Low:
  - High technology is more productive than Low technology.
  - High tech producers cannot precommit to deliver the product.
- Can choose timing of payment: before or after production.
- Renegotiation occurs if the payment is after production.
- Payment (in the decentralized market) must be in cash.
- There are residential property, \( k \).
  The private value of \( k \) is larger than the market value.
Features of the model:

- Low tech production can be implemented by simple cash payment
- High tech production cannot be implemented by simple cash payment because of the Lack of Commitment
- High tech production can be implemented by transaction using $k$ (residential property) as a hostage à la Williamson (1983)
- Disappearance of $k$ due to a financial crisis leads to the productivity declines. (Change in the production technology from High to Low.)
Policy Implications:

- Shortage of hostage-able assets (residential property) due to a financial crisis causes
  - a change in production technology from High to Low
  - the productivity declines
- Monetary injection cannot resolve the productivity declines
Plan

- Lagos-Wright model
- Partial Equilibrium in the decentralized market
  - Setup
  - Bargaining under Low tech
  - Bargaining under High tech
- Policy Implications
- General Equilibrium in the centralized and the decentralized markets
Lagos-Wight Model (1/3)

- Time is discrete. Each period is divided into day and night.
- In daytime, the decentralized market (DM) opens.
- At night, the centralized (Walrasian) market (CM) opens.
- In the DM, agents \(i\) and \(j\) meet at random. Prob that \(i\) (\(j\)) consumes what \(j\) (\(i\)) produces but not vice versa is \(\alpha\).
  (There is no double coincidence of wants.)
- Production and Preference
  - In DM, the good \(q\) is produced from labor. Utility of consuming \(q\) for buyer is \(u(q)\) and the cost of producing \(q\) for the seller is \(c(q)\).
  - In CM, each agent produce the good \(h\) from labor \(h\). Utility of consuming \(c\) units of the goods is \(U(c)\) and the cost of \(h\) is \(h\) (linear disutility)
Lagos-Wright Model (2/3)

- Bellman equation for the DM

\[ V(m) = (1 - 2\alpha)W(m) + \alpha[u(q) + W(m - d)] + \alpha[-c(q') + W(m + d)] . \]  

(1)

- Bargaining in the DM

\[ \max_{q,d} \{ u(q) + W(m_b - d) - W(m_b) \}^\theta \{ -c(q) + W(m_s + d) - W(m_s) \}^{1-\theta}, \]

(2)

subject to

\[ d \leq m_b. \]  

(3)

- Bellman equation for the CM

\[ W(m) = \max_{c,h,m+1} U(c) - h + \beta V(m_{+1}), \]

(4)

subject to

\[ c = h + \phi m - \phi m_{+1} + \tau. \]  

(5)
It is shown: $W(m) = \phi m + W_0$.

Bargaining reduces to

$$\max_{q,d} \{u(q) - \phi d\}^\theta \{-c(q) + \phi d\}^{1-\theta}, \quad \text{subject to} \quad d \leq m_b.$$ (6)

It is shown:

$$V(m) = \phi m + \alpha \{u(q(m)) - \phi d(m)\} + \alpha \{-c(q(m')) + \phi d(m')\}.$$ 

If the central bank sets $\pi = \phi/\phi_{+1}$, the real balance ($\phi m$) is determined from the envelope condition

$$V'(m) = \phi + \alpha \{u'(q(m'))q'(m) - \phi\}$$ (7) 

and the FOC wrt $m_{+1}$:

$$\phi = \beta V'(m_{+1}).$$ (8)
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Time is discrete. Each period is divided into day and night.
In daytime, the decentralized market (DM) opens.
At night, the centralized (Walrasian) market (CM) opens.
In the DM, agents $i$ and $j$ meet at random. Prob that $i$ ($j$) consumes what $j$ ($i$) produces but not vice versa is $\alpha$.
(There is no double coincidence of wants.)
Residential Property: each agent owns $k$, where the private value is $(a + x)k$ and the market value is $ak$. 
Matching technology
The agents can be separated and meet again if both of them want during the DM. (Assumption 1)

Two production technologies
- Low technology
  - Transform labor \( l \) to \( l \) units of the good
  - Production is done during the meeting of seller and buyer
- High technology (Assumption 2)
  - Transform labor \( l \) to \( Al \) units of the good, where \( A > 1 \)
  - Production must be done at home. (The seller and the buyer must be separated during the production)
Payment technology

- Agents can choose timing of payment: before or after production.
- Renegotiation occurs if the payment is after production. (Assumption 3)
- Final payment (in the decentralized market) must be in cash.

Preferences:
We assume the quasi-linear preference for the bargaining in the DM

\[ q + \phi m + (a + x) \hat{k} + ak. \]
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Bargaining under Low Tech (1/2)

If payment is before production, the bargaining is identical to Lagos-Wright.

$$\max_{q,d} \{q - \phi d\}^\theta \{\phi d - c(q)\}^{1-\theta},$$

subject to

$$d \leq m_b,$$

The bargaining outcome:

$$d = m_b,$$  \hspace{1cm} (9)

$$\phi m = z(q),$$  \hspace{1cm} (10)

where

$$z(q) \equiv \frac{\theta c(q) + (1 - \theta)c'(q)q}{\theta + (1 - \theta)c'(q)}.$$  \hspace{1cm} (11)
Bargaining under Low Tech (2/2)

- If payment is after production, down payment $e$ is necessary.
- The outcome is identical to payment before production.
  - Renegotiation after production:
    \[
    \max_{d'} \{q - \phi d'\}^\theta \{\phi d'\}^{1-\theta},
    \]  
    subject to
    \[
    d' \leq m_b - e.  \tag{13}
    \]
    The solution is
    \[
    \phi d' = (1 - \theta)q.  \tag{14}
    \]
  - Ex-ante bargaining:
    Given the solution, (14), of the renegotiation stage,
    \[
    \max_{q,d,e} \{q - \phi d\}^\theta \{\phi d - c(q)\}^{1-\theta},
    \]  
    subject to
    \[
    \phi d = (1 - \theta)q + \phi e \leq \phi m_b.  \tag{16}
    \]
Bargaining under High Tech (1/4)

- We assume $(1 - \theta)A < c'(0) < 1$. (Assumption 4)
- If cash is the sole medium of exchange, the High tech good cannot be traded. (Proposition 1)
  - Payment before production is not feasible. (The seller would abscond without producing the good.)
  - If payment is after production, seller gets at most $(1 - \theta)A l$. In this case $l = 0$ is optimal since $(1 - \theta)A - c'(0) < 0$.
  - Suppose down payment $e$ is paid to make it renegotiation-proof.
  - It is shown that $\phi e > \phi m_b - (1 - \theta)A l$.
    The incentive compatibility for seller to deliver the good is
    \[
    \phi e < \phi m_b - c(l). \quad (17)
    \]
    - Condition (17) is violated if $(1 - \theta)A l < c(l)$.
    - Assumption 4 implies that $(1 - \theta)A l < c(l)$ for all $l$. 
Bargaining under High Tech (2/4)

- Bargaining with a hostage à la Williamson (1983)
  Buyer and seller bargain over \((Al, d, k_e)\)
  
  - Renegotiation after production
    \[
    \max_d \{Al + (a + x)k_e - \phi d\}^\theta \{\phi d - ak_e\}^{1-\theta},
    \]
    subject to \(d \leq m_b\).
  
  - The solution to renegotiation
    \[
    \phi d = (1 - \theta)(Al + xk_e) + ak_e.
    \]
  
  - Ex-ante bargaining before production
    \[
    \max_{d,k_e,l} \{Al - \phi d\}^\theta \{\phi d - c(l)\}^{1-\theta}
    \]
    subject to \(\phi d = (1 - \theta)(Al + xk_e) + ak_e \leq \phi m_b,\)
    \(k_e \leq k.\)
Bargaining under High Tech (3/4)

The solution to ex-ante bargaining

\[ \phi m_b = \phi d = z(l, A), \]

(24)

\[ k_e = \frac{m_b - (1 - \theta)Al}{(1 - \theta)x + a}. \]

(25)

This solution is identical to the Lagos-Wright outcome in the case where the seller can commit to deliver the good.

The Lagos-Wright bargaining problem:

\[
\max_{l,d} \{ Al - \phi d \}^\theta \{ \phi d - c(l) \}^{1-\theta}
\]

subject to \( d \leq m_b. \)

(26)

(27)
Bargaining under High Tech (4/4)

Assumption 5

\[
\frac{x}{a} > \sup_{0 \leq l \leq l^*} G(l) > 0, \quad (28)
\]

where \( l^* \) is the solution to \( A = c'(l) \) and

\[
G(l) = \frac{\{\theta A + (1 - \theta)c'(l)\}\{c(l) - (1 - \theta)Al\}}{(1 - \theta)^2c'(l)\{Al - c(l)\}}. \quad (29)
\]

Proposition 2

The incentive compatibility condition for seller to deliver the good is satisfied by the ex-ante bargaining solution (24)–(25).

- IC for delivery is

\[
ak_e < \phi m_b - c(l), \quad (30)
\]

which can be rewritten as

\[
\frac{x}{a} > G(l(m_b)), \quad (31)
\]

where \( l(m_b) \) is the solution to (24) and \( 0 \leq l(m_b) \leq l^* \).
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A hostage asset with private value $x > 0$ is needed to implement High tech production. (Cash payment is insufficient)

Residential property can be used as a hostage.

If hostage-able assets disappear, production technology changes from High to Low. (Productivity declines)

Balance-sheet deteriorations during a financial crisis causes disappearance of the assets

- Lemon problem due to emergence of bad assets
  (Sellers no longer accept hostages because they must be lemons.)

- Excessive collateral loans
  (Buyers have no hostages to offer because all their assets have been put up as collateral for consumption loans.)
Money injection cannot resolve the problem of asset disappearance and the productivity decline.

Policy measures with fiscal outlays may be necessary
- Government purchase of bad assets
- Rehabilitation of debt-ridden borrowers through subsidies and bankruptcy procedures
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General Equilibrium (1/6)

In CM, agents can borrow and lend the consumption loans $b$, with the collateral constraint:

$$(1 + r_{+1})b_{+1} \leq a_{+1}k_{+1}. \quad (32)$$

At the beginning of DM, agents need to pay a fixed cost $\kappa$ to become able to use High tech.

There are multiple equilibria

- High equilibrium
  - All agents pay $\kappa$.
  - All agents set $b$ such that (32) does not bind and sufficient amount of $k$ is available for a hostage in DM.

- Low equilibrium
  - No agent pays $\kappa$.
  - All agents set $b$ such that (32) binds and no $k$ is available for a hostage in DM.
Agents consume only at night in CM.
In CM, each agent produce the good $h$ from labor $h$. Utility of consuming $c$ and supplying labor $h$ is $U(c) - h$.
In CM, the residential asset $k$ generates $\omega$ units of general goods and $\delta$ units of special goods (only for the original owner). Since $1 + r = \beta^{-1}$,

\[ a = \frac{\beta \omega}{1 - \beta}, \quad \text{and} \quad x = \frac{\beta \delta}{1 - \beta}. \]
General Equilibrium (3/6)

- Value function for DM: \( V(m, k, \hat{k}, b, b') \)
- Value function for CM: \( W(q, m, k, \hat{k}, b, b') \)
- Bellman equation for DM is as follows
  (Note \( k, \hat{k}, b, b' \) are omitted.):

\[
V(m) = \\
(1 - 2\alpha)W(0, m) + \alpha\hat{\sigma}W(Al'_h, m - d_h) + \alpha(1 - \hat{\sigma})W(l'_l, m - d_l) \\
+ \alpha[-c(l_l) + W(0, m + d'_l)] \\
+ \max\{-\kappa + \alpha\hat{\xi}[-c(l_h) + W(0, m + d'_h) + c(l_l) - W(0, m + d'_l)], 0\}, \quad (33)
\]
General Equilibrium (4/6)

- Bargaining under High tech (Note $b, b'$ are omitted):
  - Ex-ante Bargaining
    \[
    \max_{d_h, l_h, k_e} \{W(Al_h, m_b - d_h, k_b, \hat{k}_b) - W(0, m_b, k_b, \hat{k}_b)\}^\theta \\
    \quad \times \{-c(l_h) + W(0, m_s + d_h, k_s, \hat{k}_s) - W(0, m_s, k_s, \hat{k}_s)\}^{1-\theta},
    \]
    \[\text{(34)}\]
    subject to \(k_e \leq \hat{k}_b - \max \left\{ \frac{(1 + r)b}{a} - k_b, 0 \right\},\]
    \[d_h(l_h, k_e) \text{ is the solution to the renegotiation (35)},\]
  - Renegotiation
    \[
    \max_{d_h} \{W(Al_h, m_b - d_h, k_b, \hat{k}_b) - W(0, m_b, k_b, \hat{k}_b - k_e)\}^\theta \\
    \quad \times \{W(0, m_s + d_h, k_s, \hat{k}_s) - W(0, m_s, k_s + k_e, \hat{k}_s)\}^{1-\theta},
    \]
    \[\text{(35)}\]
    subject to \(d_h \leq m_b.\)
General Equilibrium (5/6)

- Bellman equation for CM:

\[ W(q, m, k, \hat{k}, b, b') = \max_{c, h, m_{+1}, k_{+1}, \hat{k}_{+1}, b_{+1}, b'_{+1}} U(c) - h \]
\[ + \beta V(m_{+1}, k_{+1}, \hat{k}_{+1}, b_{+1}, b'_{+1}), \]

(36)

subject to

\[ c = h + q + \omega k + (\omega + \delta)\hat{k} + \phi(m - m_{+1}) + a(k + \hat{k} - k_{+1} - \hat{k}_{+1}) \]
\[ + (1 + r)(b' - b) + (b_{+1} - b'_{+1}), \]

(37)

\[ (1 + r_{+1})b_{+1} \leq a_{+1}(k_{+1} + \hat{k}_{+1}), \]

(38)

\[ \hat{k}_{+1} \leq \hat{k}. \]

(39)
Value function for CM reduces to

$$W(q, m, k, \hat{k}, b, b') = q + (\omega + a)k + (\omega + \delta + a + x)\hat{k} + \phi m + (1 + r)(b' - b) + W_0,$$  \hspace{1cm} (40)

Bargaining problems reduce to the quasi-linear form.

Value function for DM reduces to

$$V(m, k, \hat{k}, b, b') = 
\begin{align*}
& \cdots + \max\{-\kappa + \alpha \hat{\xi}[-c(l_h) + \phi d_h + c(l_i) - \phi d_l], 0\},
\end{align*}$$  \hspace{1cm} (41)

where $\hat{\xi}$ is the prob of agents having sufficient $k$ for hostages.

- If $\hat{\xi} = 1$, agents pay $\kappa$.
- If all agents pay $\kappa$, agents reserve $k$ because hostages are valuable. Thus $\hat{\xi} = 1$. 

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