New Keynesian Dynamics in a Low Interest Rate Environment

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Some Central Questions in Economics

- How does a higher labor tax affect economic activity?

- How does an increase in neutral technology affect economic activity?

- How large is the government purchases multiplier?
Answers when the nominal rate is zero

- Labor tax ↑ → Hours ↑ → Output ↑
  (Eggertsson, 2011, “Paradox of Toil”)
- Technology ↑ → Output ↓
  (Braun and Waki, 2006, Christiano, Eichenbaum and Rebelo, 2010)
- Government purchases multiplier is well above one.
  - ≲ 4 (Christiano, Eichenbaum and Rebelo, 2010)
  - Eggertsson (2011), ≲ 2.3
  - Woodford (2011) it is unbounded as the duration of R=0 increases.
This paper’s objective

Present evidence that this focus on specifications of the NK model with unorthodox properties may be misplaced.
How are unorthodox results produced?

An abstract experiment

- Starting from a steady state with a stable price level consider a “large” shock to the preference discount factor that drives the nominal rate to zero.

- Compare these impulse responses with those from a second round where small shocks to labor taxes, technology or government purchases also arrive simultaneously.

- Strongest results arise when the duration of the second round shocks is the same or shorter than the first round shock.
How we make our case

- Investigate the empirical relevance of unorthodox results using Japanese data.

- Japan is a nice test case: Long episode of zero nominal interest rates (1999-2006).
First result

- We calibrate a NK model to Japanese data to reproduce
  - The 20% decline in output relative to trend between 1990 and 2004
  - The decline in inflation from 3.3% to -0.9% between 1991 and 2002.
  - The decline in the nominal interest rate from 7.5% to 0% between 1991 and 1999.
  - Market expectations about the expected duration of zero interest rates.

- Surprisingly the model has orthodox properties and the g multiplier is less than 1!
Preface to the second result: Evidence of tranquility

- Japan’s episode of zero nominal interest rates (1999-2005) was a period of relative tranquility: Volatilities relative to years when \( R > 0 \) (1988-1998) fall.

  - Output volatility falls by 46%
  - Inflation volatility falls by 72%
  - Real marginal cost volatility falls by 51%
Second Result

* Alter the parameterization to yield specifications that exhibit the unorthodox properties documented in the previous literature.

* Run a horse race.

* The specification with orthodox properties is consistent with the evidence of tranquility.

* The specifications with unorthodox properties are inconsistent with the evidence of tranquility.
Intuition: Why do unorthodox specifications fail?

- Need long expected duration of zero interest rates to get unorthodox results.
- Monetary policy is inactive for a long time.
- A longer period of inactive MP implies that price movements and markup response to a variety of shocks is large.
- Result is a large and counterfactual increase in economic volatility.
The model

Households

- Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t \prod_{j=0}^{t} d_j \left\{ \frac{(c_t^\nu(1 - h_t)^{1-\nu})^{1-\sigma}}{1-\sigma} \right\}, \quad d_t \sim AR(1) \]

- Budget constraint

\[ (1 + \tau_{c,t})c_t + x_t + \frac{B_t}{P_t} = (1 + R_{t-1}) \frac{B_{t-1}}{P_t} + \int_0^1 \frac{\Pi_t(i)}{P_t} di \]

\[ + T_t + (1 - \tau_{t,K})r_t k_{t-1} + (1 - \tau_{t,W})w_t h_t + \tau_{t,K} \delta k_{t-1} \]

- Capital accumulation

\[ k_t = (1 - \delta)k_{t-1} + x_t - \frac{\phi}{2} \left( \frac{x_t}{k_{t-1}} - \mu_k + 1 - \delta \right)^2 k_{t-1} \]
Firms

- Perfectly competitive final goods sector
- Imperfectly competitive intermediate goods sector
- Quadratic price adjustments costs
Final Goods Sector

\[ y_t = \left( \int_0^1 y_t(i) \frac{\theta - 1}{\theta} \, di \right)^{\frac{\theta}{\theta - 1}} \]  \hspace{1cm} (1)

The profit maximizing input demands of the final good firm are

\[ y_t(i)^d = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} y_t \]  \hspace{1cm} (2)
Intermediate Goods Sector

Production technology for producer $i$

$$y_t(i) = k_{t-1}(i)^\alpha (A_t h_t(i))^{1-\alpha}$$

(3)

Cost minimization implies

$$r_t = \alpha \chi_t k_{t-1}(i)^{\alpha-1} (A_t h_t(i))^{1-\alpha}$$

(4)

$$w_t = (1 - \alpha) \chi_t A_t^{(1-\alpha)} k_{t-1}(i)^\alpha h_t(i)^{-\alpha}$$

(5)
Price setting problem

Maximize by choice of $p_t(i)$

$$\sum_{t=0}^{\infty} \beta^t \prod_{j=0}^{t} d_j \lambda_{c,t} \left[ p_t(i)y_t(i) - P_t s \chi_t y_t(i) - \frac{\gamma}{2} P_t (\pi_t(i) - \pi)^2 y_t \right] / P_t \quad (6)$$
Fiscal and Monetary Policy

- Lump-sum taxes adjust to satisfy the government budget constraint.
- Monetary policy follows a Taylor-type rule

$$R_t = \max[(1 + R) \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\rho_{\pi}} \left( \frac{1 + R_{t-1}}{1 + R} \right)^{\rho_P} e^{u_{M,t}} - 1, 0]$$

NK Dynamics when $R=0$

April 28, 2011 16 / 30
Aggregate resource constraint

In the model there is a distinction between $GDP_t$ (output) and production ($y_t$).

\[
GDP_t \equiv g_t + c_t + x_t = y_t \left(1 - \frac{\gamma_0}{2} (\pi_t)^2 \right) = y_t (1 - \Psi_t)
\]
Equilibrium

- **Perfect Foresight Equilibrium** Given a sequence of shocks and initial conditions we can solve for a sequence of prices and allocations in the standard way.

- **Definition is Incomplete** without specifying the periods with a binding constraint on the nominal rate.

- We limit attention to equilibria in which the constraint binds from $T_1$ to $T_2$ and is positive in all periods after $T_2$.

- **Stochastic Equilibrium**
  1. Solve for perfect foresight equilibrium, assuming current period shocks are nonzero and all future shocks are zero. Keep the period 1 prices and allocations. Toss the rest.
  2. Step forward one period. New shocks arrive.
  3. repeat 1.
Taking the model to the data

- Global solution method “Extended Shooting” or “Extended Path” (see e.g. Heer and Maussner (2008) or Adjemian and Juillard (2010)) (Advantage: It can handle a large number of state variables).

- Most of the previous literature linearizes all equilibrium conditions other than the zero bound constraint.

- Model is calibrated

- Preference shocks are calibrated such that the zero lower bound binds from 1999 to 2006.
Calibration

- Model period 1 year
- Parameters are calibrated.
- Calibration similar to Braun and Waki (2006).
  - Adjustment costs on prices: 80 (0.75)
  - Adjustment cost on capital: 4
- Most shocks follow AR 1 with 0.9 serial correlation.
- Taylor rule $\rho_\pi = 1.7$.
- Allow for persistence in Taylor rule. Lagged coefficient on nominal rate is 0.4.
- Preference parameters $\beta = 0.9995$, $\sigma = 2$, $\nu = 0.4$. 
Deriving Shocks

Start with shocks from Japanese data.

a) Permanent technology shocks.

b) Government purchase shocks.

c) Labor tax shocks.

d) Capital tax shocks.

e) Use consumption tax to reproduce path of labor input through 1992.
First result: NK model calibrated in this way produces orthodox results.

*Impact response of GDP and Marginal Cost.*

<table>
<thead>
<tr>
<th>Year</th>
<th>1995</th>
<th>1999</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years expected nominal rate is zero</td>
<td>none</td>
<td>1999-2000</td>
<td>2004-2005</td>
</tr>
<tr>
<td>Resource costs of price adjustment**</td>
<td>0.22</td>
<td>0.59</td>
<td>0.54</td>
</tr>
</tbody>
</table>

**shock in:**
- Neutral technology (transitory) | 0.64 | 0.57 | 0.59 |
- Neutral technology (permanent) | 0.62 | 0.68 | 0.71* |
- Labor tax | -0.62 | -0.56 | -0.57* |
- Government purchases | 0.65 | 0.87 | 0.87* |

**in:**
- Neutral technology (transitory) | 0.06 | 0.21 | 0.19 |
- Neutral technology (permanent) | -0.03 | -0.14 | -0.17* |
- Labor tax | -0.06 | -0.23 | -0.21* |
- Government purchases | -0.20 | -0.64 | -0.57* |

*For this shock the zero bound constraint applies only in 2004

**Resource costs of price adjustment are reported in percentage terms of output
How to generate unorthodox results

- Higher persistence in d shock $0.9 \rightarrow 0.94, 0.95$, holding fixed the shocks.

- Larger d shocks, holding fixed the persistence.

- Both strategies increase the expected duration of zero interest rates.
Different types of unorthodox results

- **Orthodox and g mult. < 1**: Calibrated expected duration of $R = 0$ (2 years).

- **Orthodox and g mult > 1**: Longer expected duration of $R = 0$ (4-5 years).

- **Unorthodox and g multiplier > 1**: Very long expected duration of $R = 0$ (6-7 years).
Impulse responses, shock arrives in 1999

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource costs of price adjustment*</td>
<td>0.59</td>
<td>1.53</td>
<td>3.62</td>
<td>2.51</td>
<td>7.08</td>
<td>18.70</td>
</tr>
<tr>
<td>Impact response of output (GDP) to a positive shock in:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutral technology</td>
<td>0.57</td>
<td>0.08</td>
<td>-0.38</td>
<td>0.29</td>
<td>-0.04</td>
<td>-0.39</td>
</tr>
<tr>
<td>Labor tax</td>
<td>-0.56</td>
<td>-0.08</td>
<td>0.33</td>
<td>-0.28</td>
<td>0.05</td>
<td>0.42</td>
</tr>
<tr>
<td>Government purchases</td>
<td>0.87</td>
<td>1.55</td>
<td>1.92</td>
<td>1.33</td>
<td>1.70</td>
<td>2.02</td>
</tr>
</tbody>
</table>

*Resource costs of price adjustment are reported in percentage terms of output.

For the specification without price adjustment costs in the resource constraint, implied resource costs are reported.
## Empirical Relevance

### I. Which specifications are most consistent with Japan’s evidence of tranquility?

<table>
<thead>
<tr>
<th>Variable</th>
<th>Japanese Data</th>
<th>Flexible Price</th>
<th>Moderate Price adjustment cost</th>
<th>Baseline</th>
<th>Higher discount factor serial correlation (0.94)</th>
<th>Higher discount factor serial correlation (0.95)</th>
<th>Persistent Expectations</th>
<th>Large Preference Shock</th>
<th>Large Preference Shock (alt)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.52</td>
<td>0.4</td>
<td>0.41</td>
<td>0.50</td>
<td>0.74</td>
<td>1.40</td>
<td>1.09</td>
<td>2.14</td>
<td>2.84</td>
</tr>
<tr>
<td>Output</td>
<td>0.28</td>
<td>0.69</td>
<td>0.71</td>
<td>0.75</td>
<td>1.05</td>
<td>1.36</td>
<td>1.36</td>
<td>1.87</td>
<td>2.16</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.33</td>
<td>0.38</td>
<td>0.44</td>
<td>0.53</td>
<td>0.47</td>
<td>0.65</td>
<td>1.14</td>
<td>1.8</td>
<td>5.64</td>
</tr>
<tr>
<td>Labor Input</td>
<td>0.51</td>
<td>0.00</td>
<td>0.15</td>
<td>0.21</td>
<td>1.86</td>
<td>2.25</td>
<td>1.84</td>
<td>3.01</td>
<td>5.56</td>
</tr>
<tr>
<td>Real Marginal Cost</td>
<td>0.35</td>
<td>1.02</td>
<td>0.27</td>
<td>0.07</td>
<td>1.09</td>
<td>1.68</td>
<td>0.78</td>
<td>1.47</td>
<td>2.46</td>
</tr>
<tr>
<td>Consumption deflator</td>
<td>0.35</td>
<td>0.29</td>
<td>0.25</td>
<td>0.29</td>
<td>0.13</td>
<td>0.11</td>
<td>0.62</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td>CPI</td>
<td>-</td>
<td>0.11</td>
<td>0.11</td>
<td>0.14</td>
<td>0.14</td>
<td>0.28</td>
<td>0.28</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>Preference discount shock</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Consumption tax shock</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*All statistics are calculated as the standard deviation of the variable from 1999 to 2006 relative to its standard deviation from 1988 to 1998. All variables except marginal cost are expressed in terms of log growth rates. (alt) refers to the specification where the resource costs of price adjustment are omitted from the resource constraint.*
Empirical Relevance

II. Expected Duration of the Zero Interest Rate Episode

- In the baseline specification, agents expect a binding zero lower bound for two years using estimates of Ichiue and Ueno (2007)

- The expected duration in the unorthodox specifications is counterfactually long (4 and 7 years)
Empirical Relevance

III. What is driving the unorthodox results? Example of G-multiplier

<table>
<thead>
<tr>
<th>Specification</th>
<th>( \frac{\partial GDP}{\partial g} ) = ((1 - \psi) x (\frac{\partial y}{\partial g}) - y x (\frac{\partial \Psi}{\partial g}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.87 (= (1 - 0.006) \times 0.78 - 1.02 \times (-0.09))</td>
</tr>
<tr>
<td>Discount factor serial correlation (0.94)</td>
<td>1.55 (= (1 - 0.015) \times 1.03 - 1.03 \times (-0.52))</td>
</tr>
<tr>
<td>Discount factor serial correlation (0.95)</td>
<td>1.92 (= (1 - 0.036) \times 0.87 - 1.03 \times (-1.05))</td>
</tr>
<tr>
<td>Persistent expectations</td>
<td>1.33 (= (1 - 0.025) \times 0.87 - 1.00 \times (-0.46))</td>
</tr>
<tr>
<td>Large preference shock</td>
<td>1.70 (= (1 - 0.071) \times 0.67 - 0.99 \times (-1.03))</td>
</tr>
</tbody>
</table>

- The reason for the large \(g\) multiplier is not higher production.
- The reason is lower resource costs of price adjustment: more output available for consumption and investment.
- Similar decompositions indicate that production falls when the labor tax is increased and production increases when technology improves.
Empirical Relevance

IV. Resource Costs of Price Adjustment

- Levy et al (1997) menu costs about 0.7% of revenues for supermarket chains.

- In the baseline specification, resource costs are (0.6% of GDP)

- In the other specifications, resource costs are implausibly large (1.5 ~ 7% of GDP)
Solution Method

- Loglinear method works poorly in the unorthodox scenarios (implied resource costs are 19% of GDP for large preference shock) (see also Braun, Körber and Waki (2011))
- Linearization of the resource constraint is a particularly big problem.
- Unorthodox response to labor and technology shock is smaller if global method is used.
- Using a global method also lowers the size of the government purchases multiplier.
- Recognizing the resource costs of price adjustment stabilizes the economy. See Braun, körber and Waki (2011) for more details).
Concluding Remarks

- We find that an empirically relevant specification of a prototypical NK model has orthodox properties.
- Specifications that produce unorthodox responses are not empirically relevant:
  1. Inconsistent with the evidence of tranquility in Japan.
  2. Require long expected durations of a zero interest rates that are inconsistent with market data.
  3. Big menu costs: the savings from small reductions in the resource costs of price adjustment are very large. Production and output move in opposite directions.
  4. Implausibly large resource costs of price adjustment.

These are the reasons that lead us to believe that the focus on specifications of the NK model with unorthodox properties may be misplaced.